Metamaterial-based Electromagnetic Space, Time & Space-Time Dispersion Engineering

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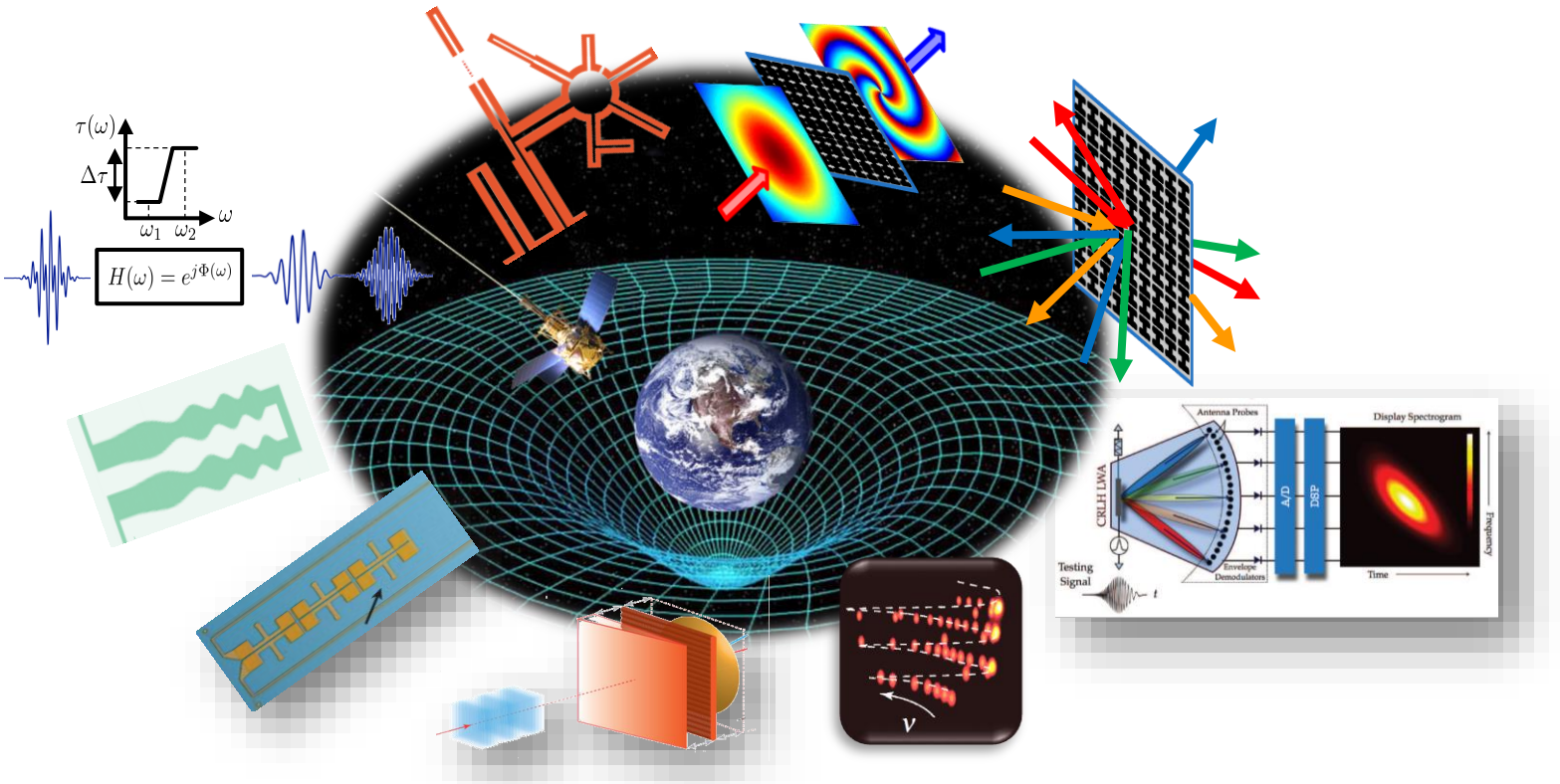

Abstract

Everything in our universe occurs in space, time, and spacetime where space and time are interdependent. These concepts are therefore fundamental across all areas of human activities, including history, economy, philosophy, arts and sciences.

The author believes systematic endeavours in manipulating waves in space, time and spacetime will bring about considerable opportunities towards the development of tomorrow's electromagnetic science and technology.

Manipulation waves in space essentially consists in engineering their spatial frequency (k) spectrum, which may be performed using conventional electromagnetic structures, such as apertures, antennas, lenses, polarizers, photonic crystals, Talbot imagers, optical masks, etc. However, the frontiers of spatial dispersion engineering may be pushed far beyond the current state of the art, particularly using the novel concept of metasurfaces. The slides present several innovations in this area, including magnetless nonreciprocal gyrotropy, generalized refraction, multiple wave transformation, multi-refringence, and orbital angular momentum multiplexing.

In time, manipulating waves essentially consists in engineering their temporal frequency (ω) spectrum, as partly done in ultrafast optics where oscillations are too fast to be handled by digital signal processors, and where real-time chirping and nonlinear materials and devices have therefore to be used instead. Such concepts have been little explored in electromagnetics, and may represent a solution to the exploding demand for faster and more reliable radio if sufficient progress is made. The author developed in his group metamaterial-inspired structures called phasers, which provide specifiable group delay versus frequency responses to perform unprecedented temporal dispersion engineering. The slides present the related Radio Analog Signal Processing (R-ASP) concept and technology, and demonstrate a number of related applications, such as spectrum sniffing, real-time Fourier analysis, and dispersion code multiple access.

Aforementioned concepts typically concern monochromatic spatial dispersion engineering and mono-directional temporal dispersion engineering. Combining the two aspects in simultaneous space and time dispersion engineering, as nature does it in rainbows and humans in holograms, will open up further horizons in electromagnetics processing. The slides also address this area, first introducing the fundamental concept of temporal to spatial frequency mapping, and next describing a few recent applications, including real-time spectrogram analysis and two-dimensional mapping for the processing of ultrafast waves.

Finally, novel concepts in electromagnetic spacetime discontinuities, where space and time are interdependent as in the theories of special and general relativity, will be introduced and discussed as a potential platform for future developments in electrodynamics.

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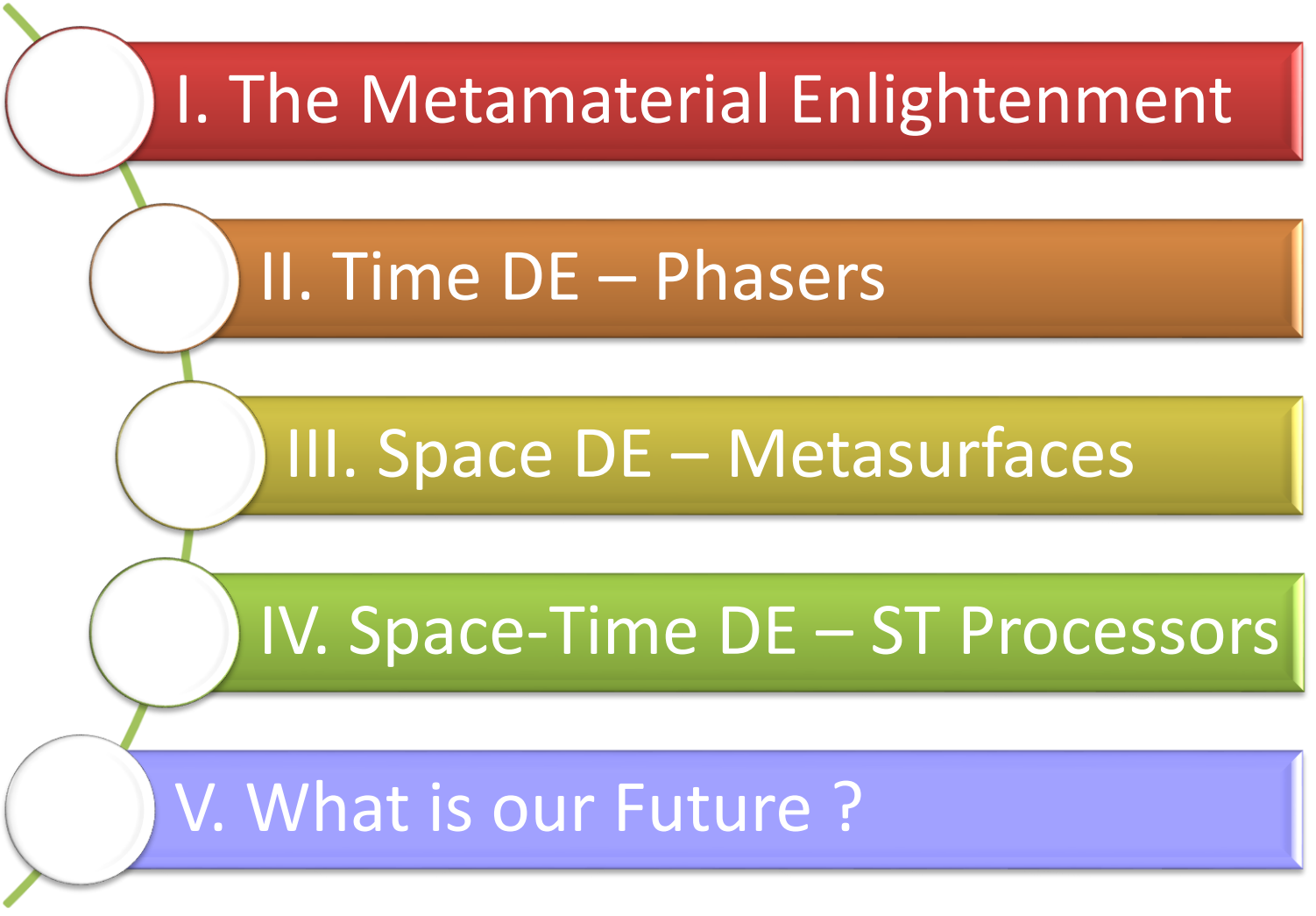
Outline


I. The Metamaterial Enlightenment
II. Time DE – Phasers
III. Space DE – Metasurfaces
IV. Space-Time DE – ST Processors
V. What is our Future ?

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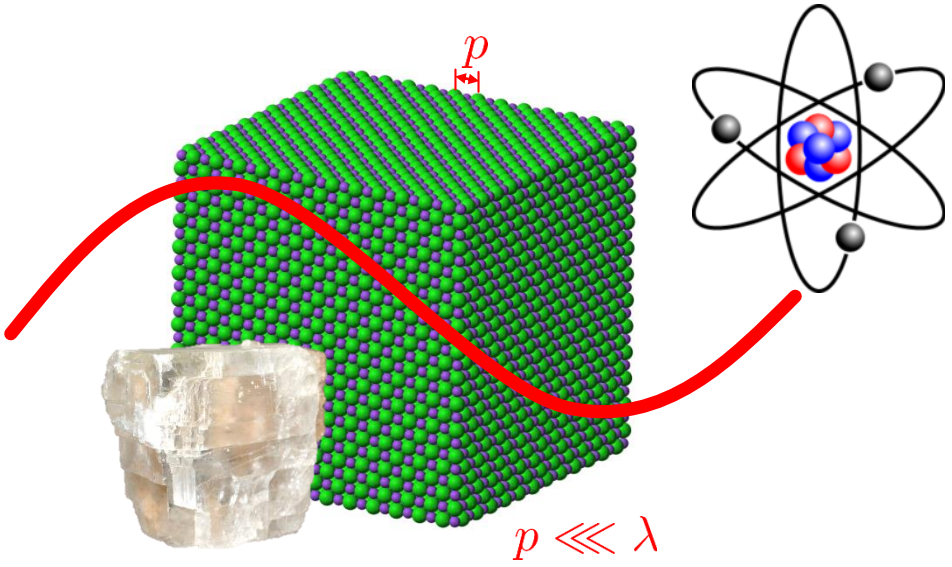
Metamaterial (MTM) Definition and Motivation

Material:

arrangement of atoms

with properties

due to the molecular scale



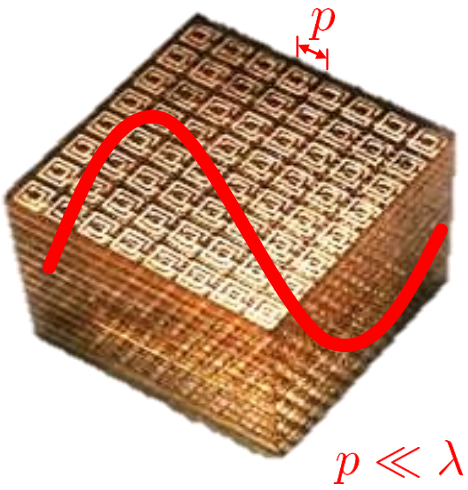
more ↓ properties

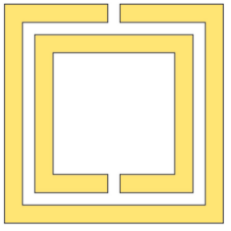
Metamaterial (MTM): (μετά = "beyond")

arrangement of scatterers

with engineerable extra properties

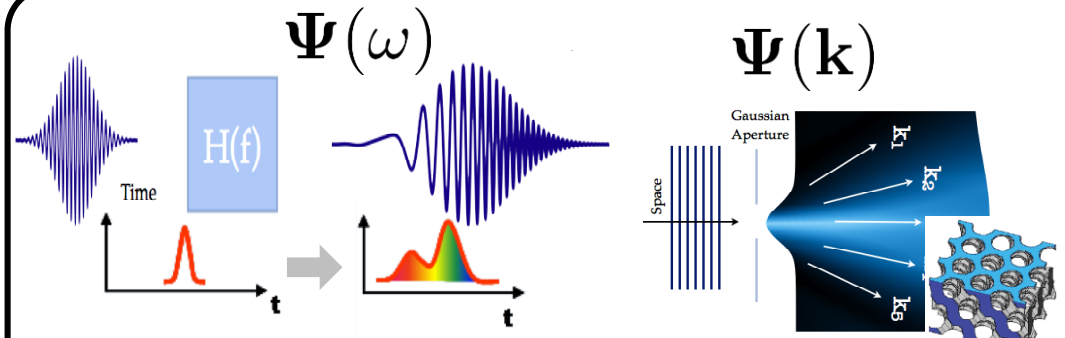
due to a higher structural scale





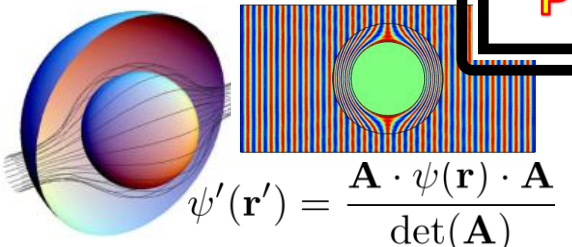
Properties: Constitutive Parameters

Diversity of Engineerable Properties

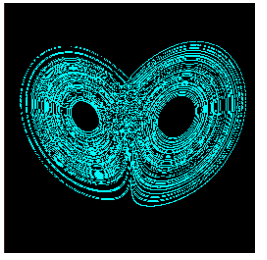


temporal dispersion

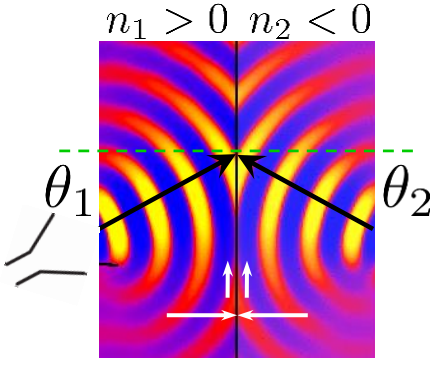
spatial dispersion



graded index



nonlinearities

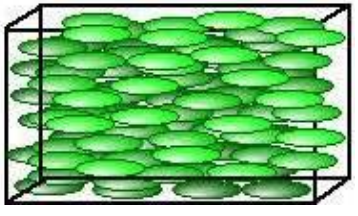


negative refraction

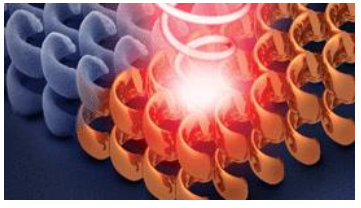
$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\mathbf{D} = \bar{\bar{\varepsilon}}(\omega, \mathbf{k}, \mathbf{r}, |E|, |H|) \cdot \mathbf{E} + \bar{\bar{\xi}}(\omega, \mathbf{k}, \mathbf{r}, |E|, |H|) \cdot \mathbf{H}$$

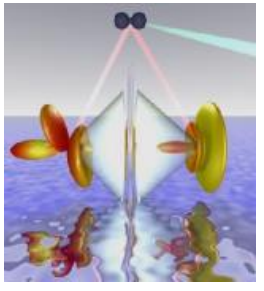
$$\mathbf{B} = \bar{\bar{\zeta}}(\omega, \mathbf{k}, \mathbf{r}, |E|, |H|) \cdot \mathbf{E} + \bar{\bar{\mu}}(\omega, \mathbf{k}, \mathbf{r}, |E|, |H|) \cdot \mathbf{H}$$



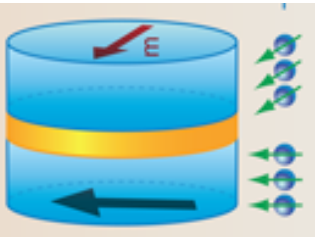
anisotropy



bi/bian-isotropy



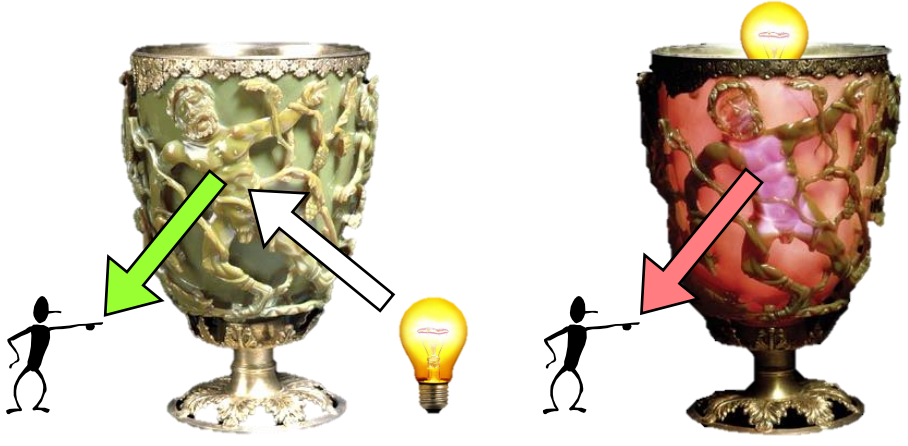
nano-functionality



quantum effects

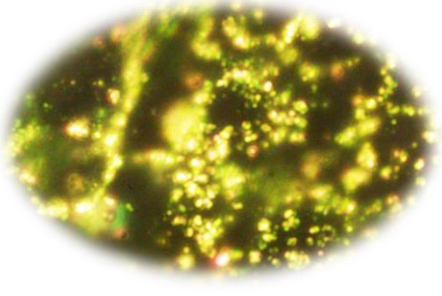
Pre-History (BC – XIXth Century)

Lycurgus Cup
4th century AD

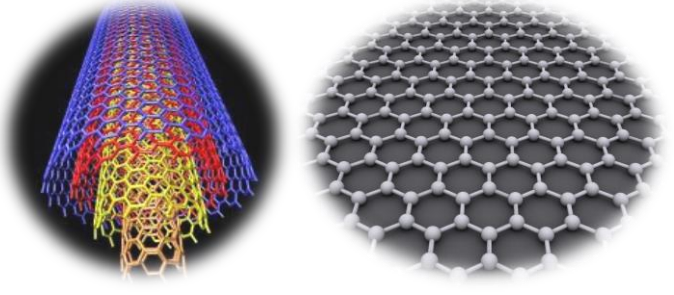


Damascus Steel
1100-1700 AD

gold-silver nanoparticles



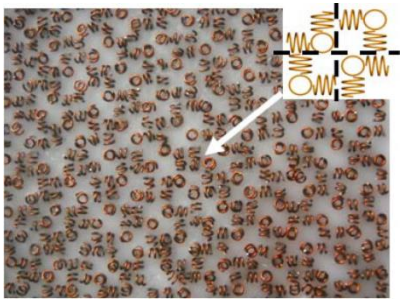
carbon nanotubes / graphene



Recent History (XXth Century)

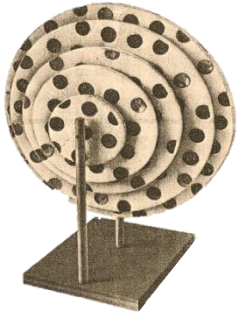
Tinfoil-paper Polarizer
1898, J. C. Bose

Chiral Medium
1920, K. F. Linddman

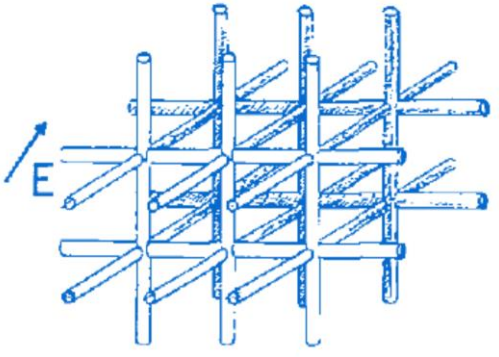


Artificial Dielectrics

Light Lens
1949, W. E. Kock



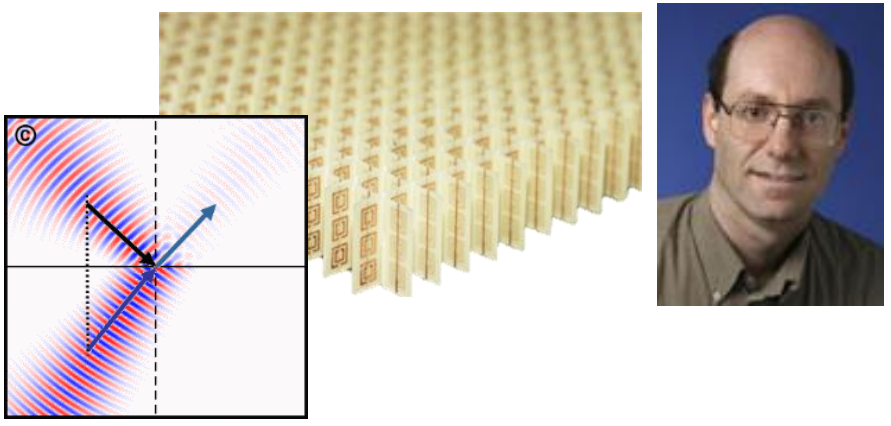
Artificial Plasma
1962, W. Rotman



Last 15 years' Boom

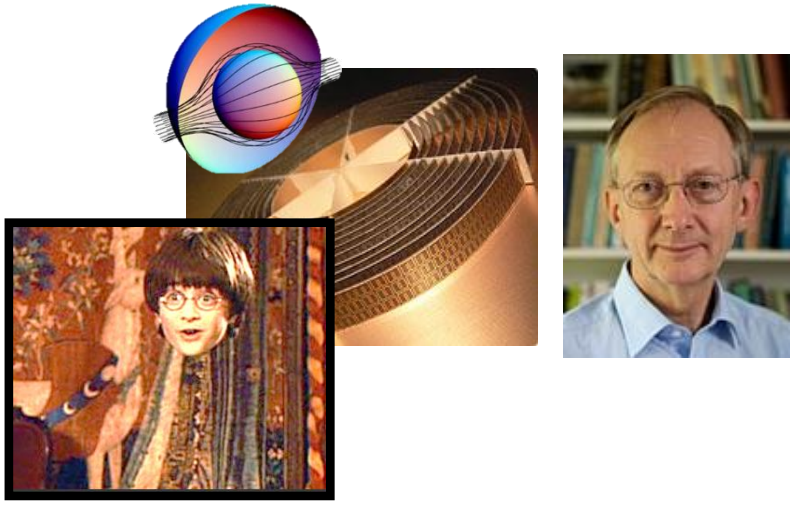
Negative Refractive Index

2000, D. R. Smith *et al.*



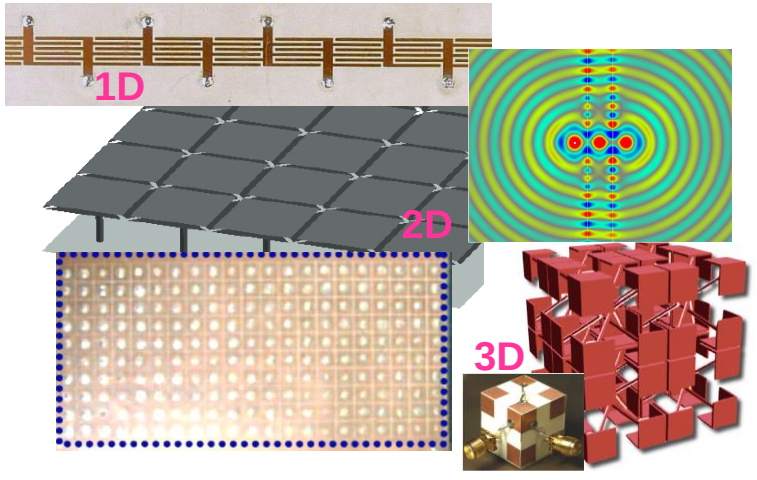
Transformation Optics

2006, J. Pendry *et al.*



Transmission-Line MTMs & Apps

2001-2002, Caloz & Itoh; Oliner, Eleftheriades



Applications 2002 – …

- **Super-resolution imaging / cloaking**
- **Enhanced EM devices**
  bandwidth, coupling, multiband, diplexing, feeding, non-reciprocity, etc.
- **Agile antenna/scattering systems**
  resonant/leaky-wave antennas, radomes, reflectors, beam formers, DOA, etc.
- **Real-time Analog Signal processing**
  RT Fourier Analyzers, DCMA, etc.

Temporal Dispersion, NRI Example

Plane Wave

$$\mathbf{E} = \mathbf{E}_0 e^{-j\mathbf{k}\cdot\mathbf{r}}$$

$$\mathbf{H} = \frac{\mathbf{H}_0}{\eta_0} e^{-j\mathbf{k}\cdot\mathbf{r}}$$

Maxwell Equations

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E}$$

$$\nabla = -j\mathbf{k}$$

$$\mathbf{k} \times \mathbf{E} = \omega\mu\mathbf{H}$$

$$\mathbf{k} \times \mathbf{H} = -\omega\varepsilon\mathbf{E}$$

Energy Relations

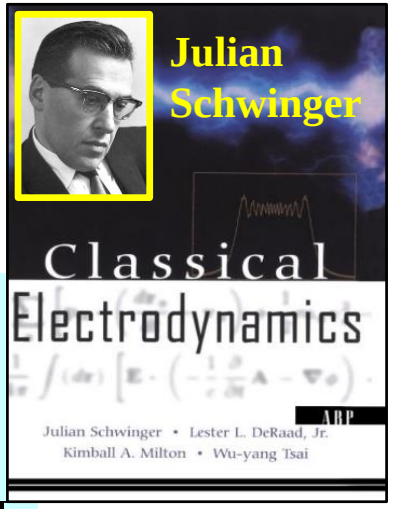
$$\bar{W} = \frac{1}{4}\left[\frac{d(\omega\varepsilon)}{d\omega}\mathbf{E}^2 + \frac{d(\omega\mu)}{d\omega}\mathbf{H}^2\right] > 0$$

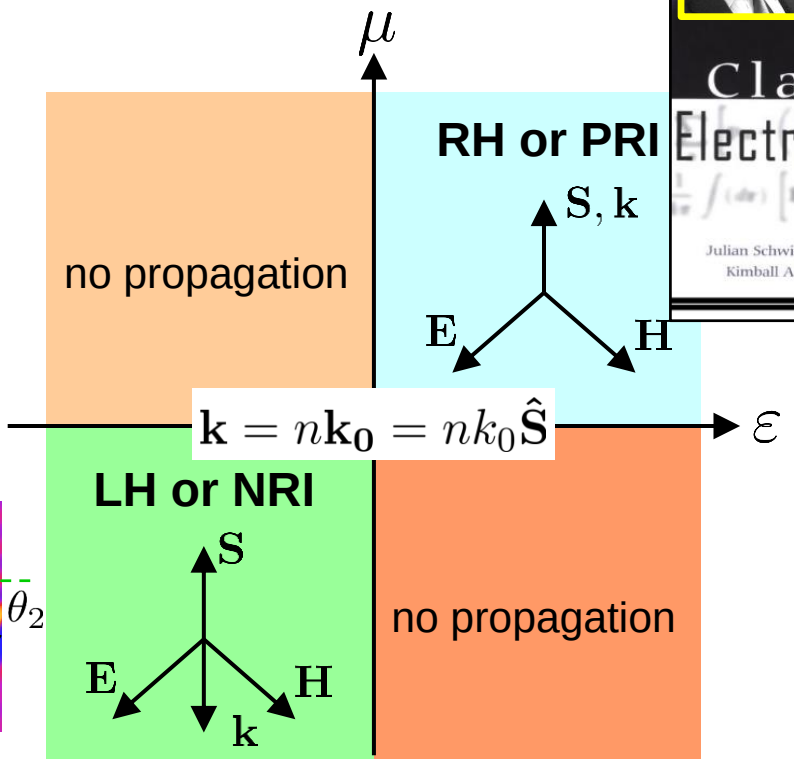
$$\frac{d(\omega\varepsilon)}{d\omega} > 0, \frac{d(\omega\mu)}{d\omega} > 0$$

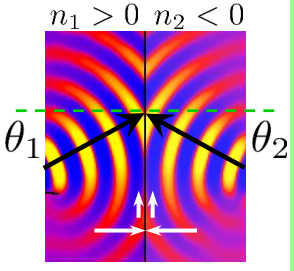
Frequency Dispersion

$$\varepsilon = \varepsilon(\omega)$$

$$\mu = \mu(\omega)$$



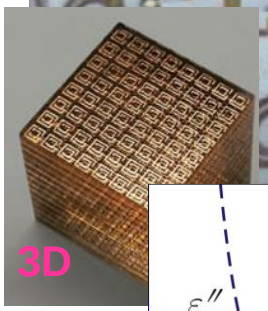


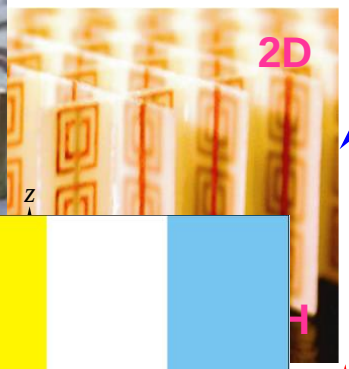


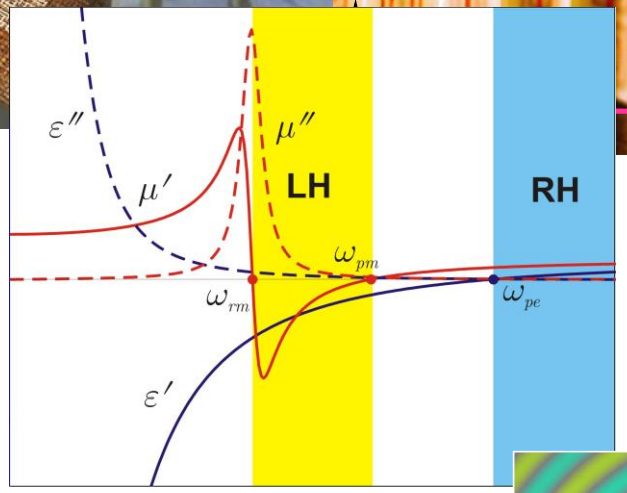
Resonant Particle (RP) versus Transmission Line (TL) MTMs

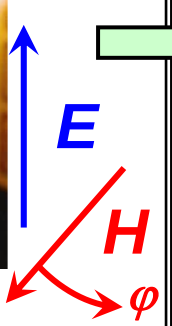
Resonant Particle (RP)
(~ volumetric)

Smith et al., 2000-2



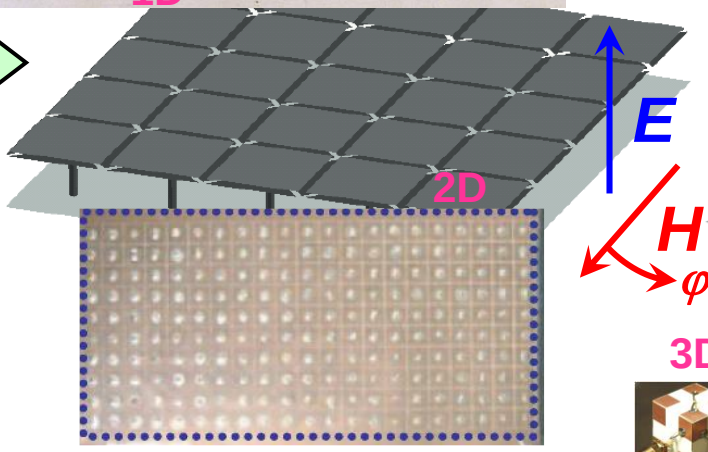


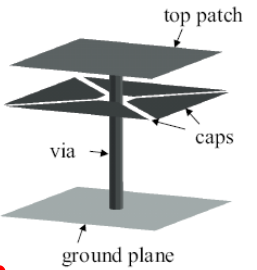


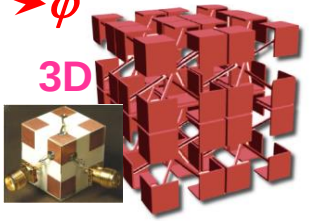


Transmission Line (TL)
(~ planar)

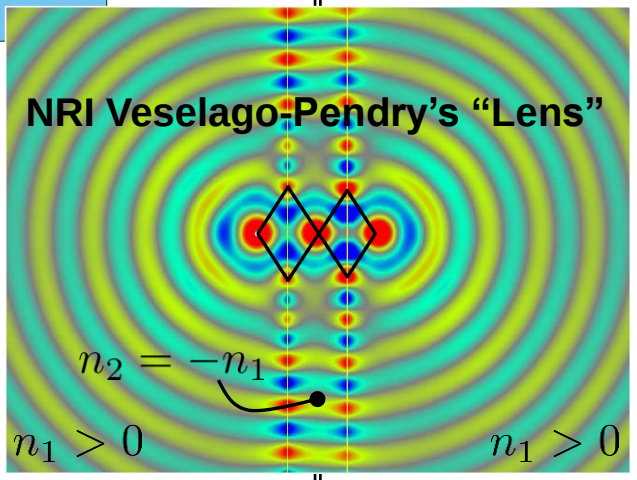
Caloz et al., 2001-3







Tighly coupled "Particles"



☒ **Narrow band**

☒ **Highly lossy**

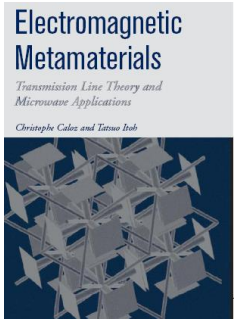
☒ **Bulky**

☑ **Broad band**

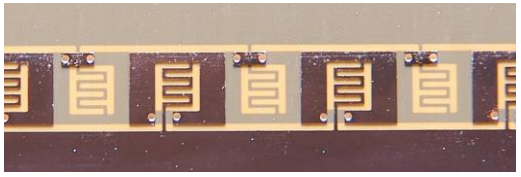
☑ **Low Loss**

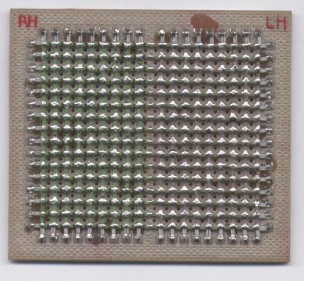
☑ **Planar – MMIC**

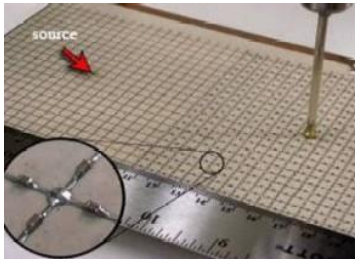
☑ **Dispersion engineering**

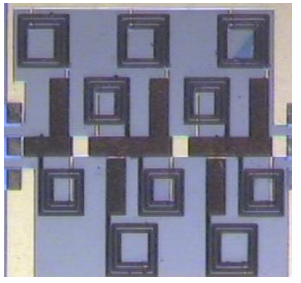


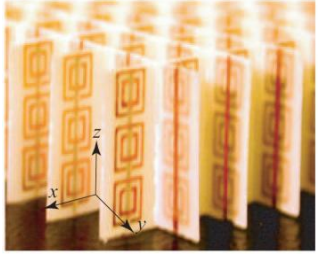
Electromagnetic Metamaterials

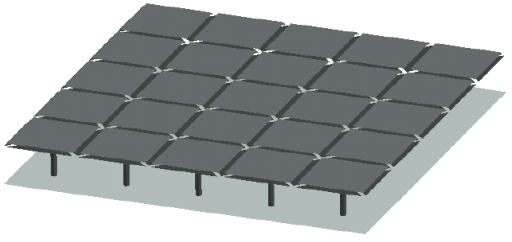


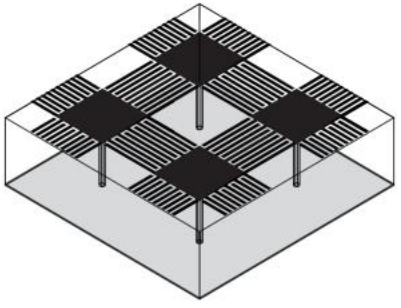


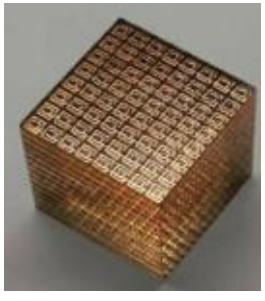


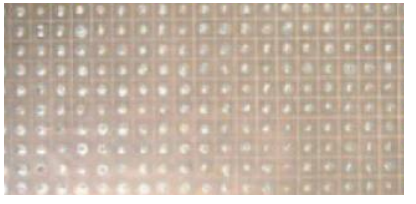


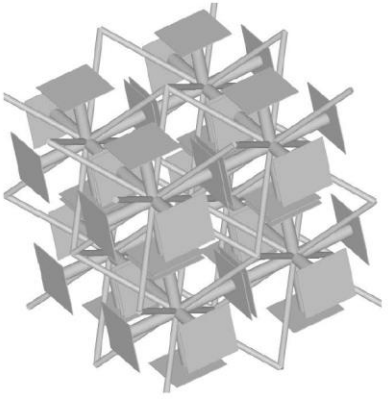


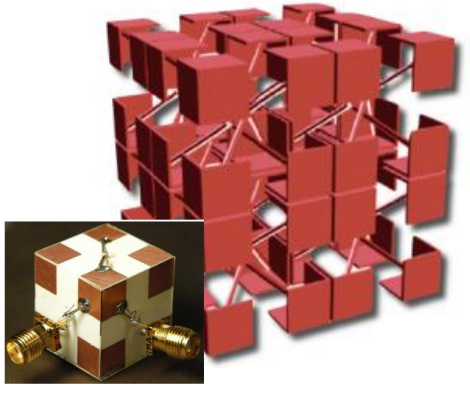












The Metamaterial (MTM) QUIZ

1) **MTMs are artificial materials with NRI.**

   NOT ONLY. Wealth of + properties & applications

2) **MTMs are a new concept of the 21st century.**

   NO, old craft structures & artificial dielectrics

   YES, novel properties & innovation paradigm

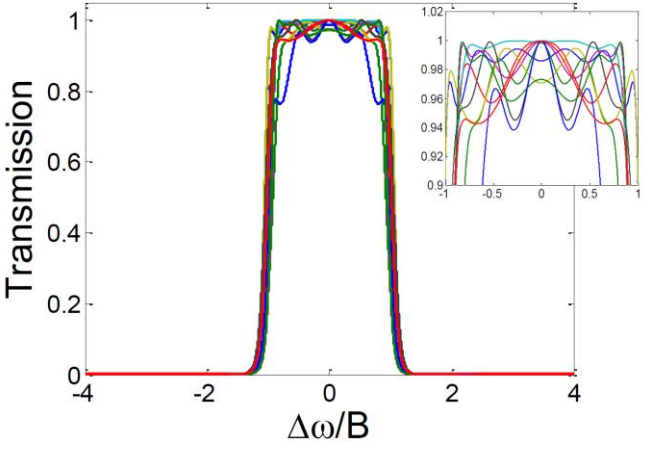
3) **RP & TL MTMs are fundamentally different.**

   NO, both made of electric and magnetic dipoles

   YES, TL MTM = coupled system => unlimited bandwidth

4) **MTMs lead to > 100 practical applications.**

   YES, mostly in RF range



5) **MTMs tend to be highly dispersive.**

   YES, consequence of causality (KK relations)

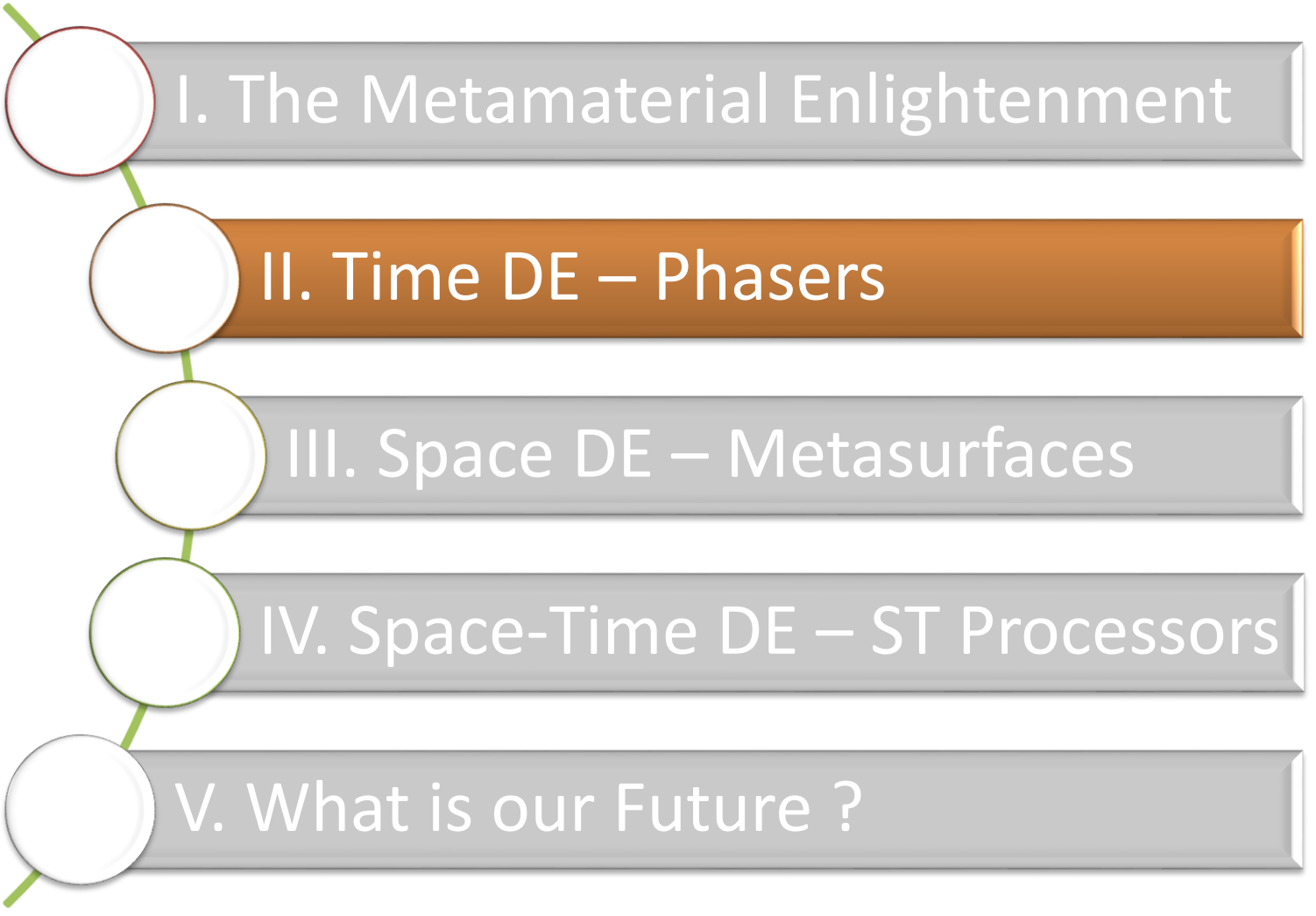
6) **Dispersion is a major issue because of distorsion.**

   YES, if uncontrolled. NO if leveraged !

7) **Why does this weird guy consider MTMs an enlightenment?**

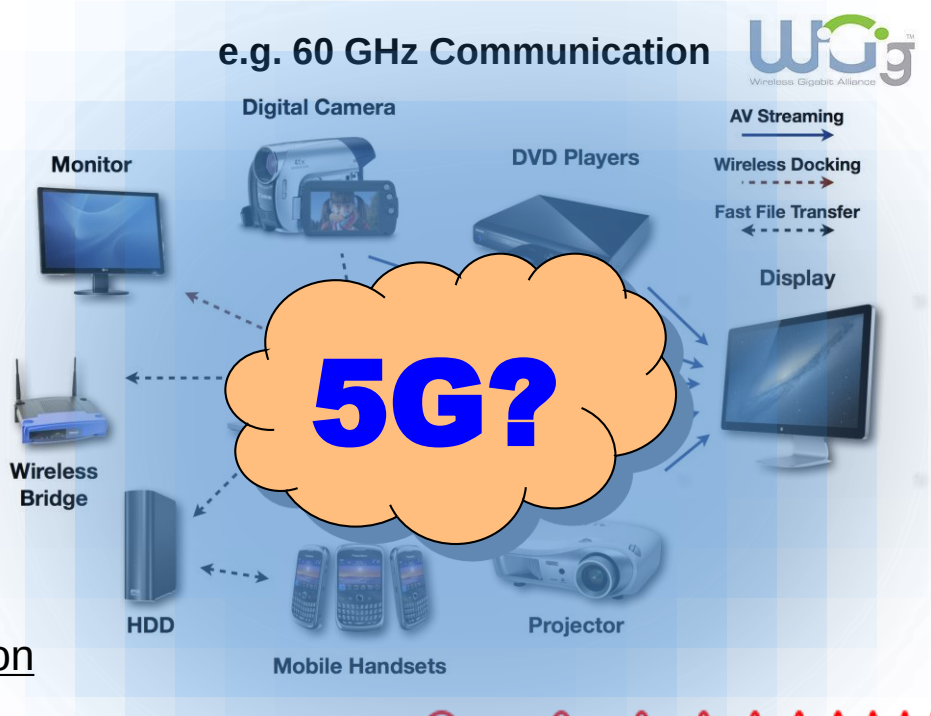
   Dispersion Engineering = new approach of EM science & technology

Outline


I. The Metamaterial Enlightenment
II. Time DE – Phasers
III. Space DE – Metasurfaces
IV. Space-Time DE – ST Processors
V. What is our Future ?

Motivation

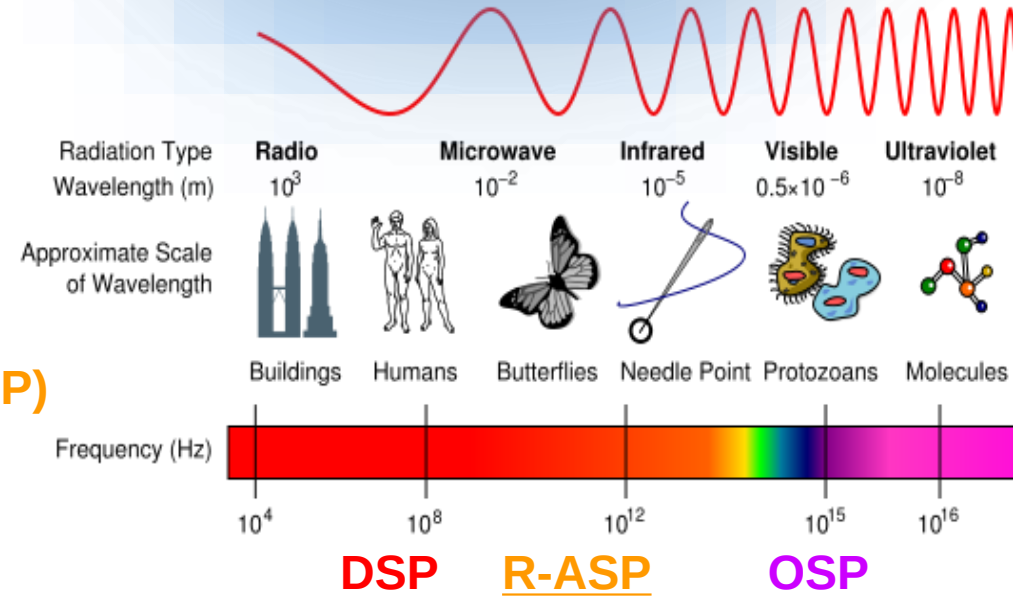
- **Exploding demand for faster & more reliable & ubiquitous EM technology** in communications, instrumentation, radar, imaging, sensing, etc.



- **Digital Signal Processors (DSP)**
  - ☺ high flexibility, reliability & compactness
  - ☹ buffering & high data rates
    - ⇒ limited speed & high ADC power consumption

- **Optical Signal Processing (OSP)**
  - ☺ analog, real-time ⇒ high speed
  - ☹ EM dispersion ⇒ limited control

- **Radio Analog Signal Processing (R-ASP)**
  - ☺ real-time ⇒ high speed & low power
  - ☺ new DE technology ⇒ high flexibility



C. Caloz, S. Gupta, Q. Zhang, and B. Nikfal, *IEEE Microw. Mag.*, vol. 14, no. 6, pp. 87-103, Sept. 2013.

R-ASP: Dispersion – Group Delay – Engineering

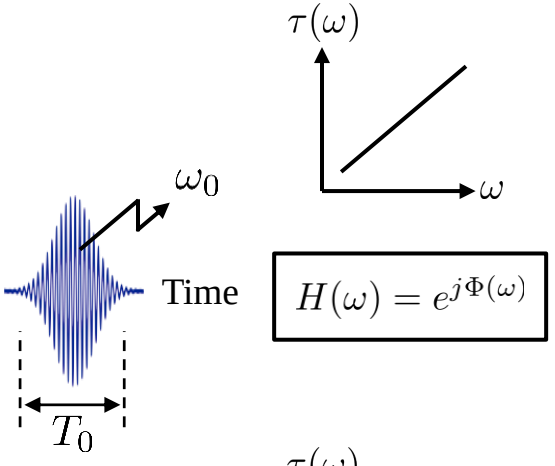
Nonlinear phase:

$$\Phi(\omega) = \Phi_0 + \Phi_1(\omega-\omega_0) + \frac{\Phi_2}{2}(\omega-\omega_0)^2 + \frac{\Phi_3}{6}(\omega-\omega_0)^3 \ldots$$

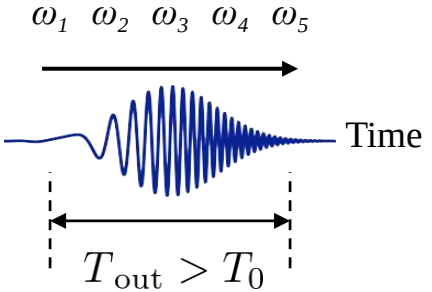
$$\Phi_n = \left.\frac{\partial^n \Phi(\omega)}{\partial \omega^n}\right|_{\omega=\omega_0}$$

Dispersive delay:

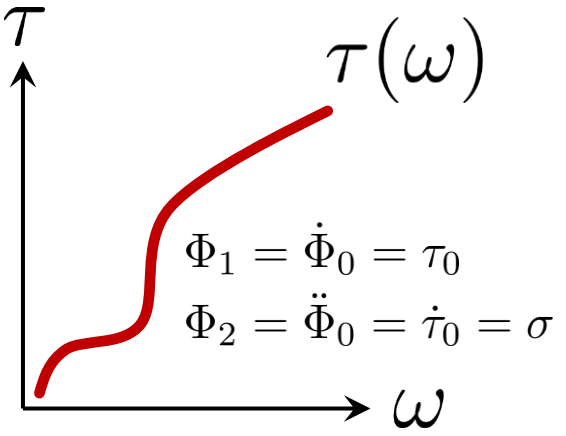
$$\tau(\omega) = \frac{d\Phi(\omega)}{d\omega} = \Phi_1 + \Phi_2(\omega-\omega_0) + \frac{\Phi_3}{2}(\omega-\omega_0)^2 \ldots$$

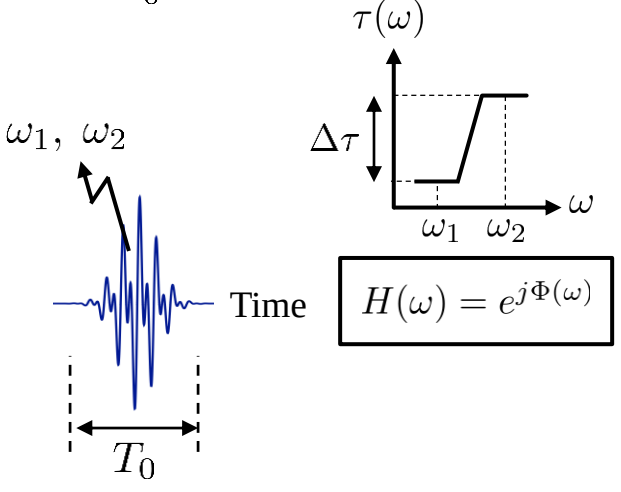


Frequency Chirping

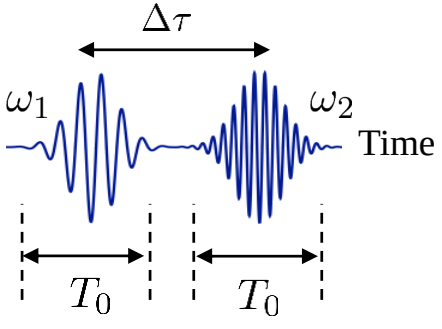


Group Delay Engineering

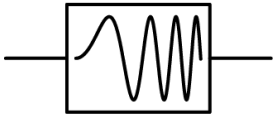




Frequency Discrimination



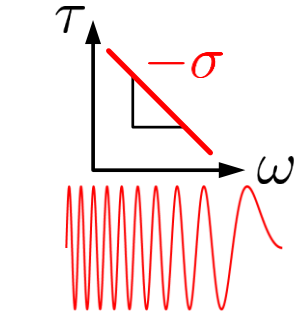
PHASER



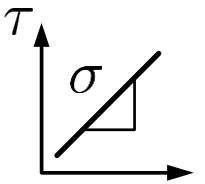
Real-time signal processing using group delay engineered analog (EM) structures

Real-Time Fourier Transformation (RTFT)

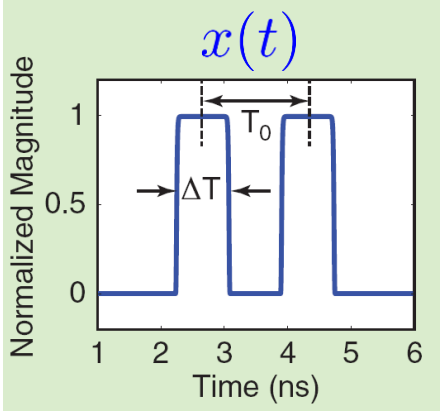
$$c(t) = \exp\left(-j\frac{t^2}{2\sigma}\right)$$

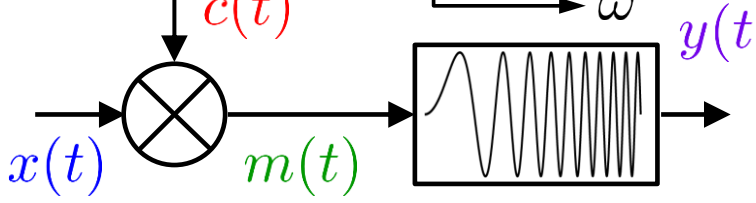


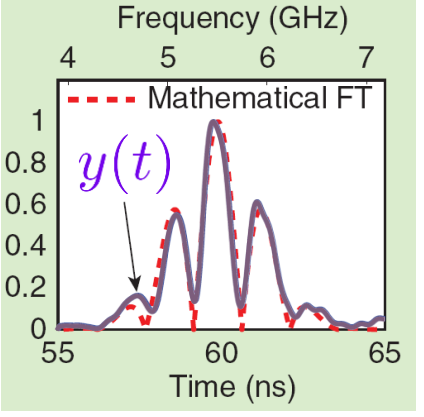
$$h(t) = \exp\left(j\frac{t^2}{2\sigma}\right)$$



$$|y(t)| = \int_{-\infty}^{+\infty} x(\tau)\exp\left[-j\omega(t)\tau\right]\,d\tau$$







$$m(t)=x(t)c(t)=x(t)\exp\left(-j\frac{t^2}{2\sigma}\right)$$

$$y(t)=m(t)*h(t)=\int_{-\infty}^{+\infty} x(\tau)\exp\left(-j\frac{\tau^2}{2\sigma}\right)\exp\left[j\frac{(t-\tau)^2}{2\sigma}\right]d\tau$$

$$=\exp\left(j\frac{t^2}{2\sigma}\right)\int_{-\infty}^{+\infty} x(\tau)\exp\left(j\boxed{\frac{t}{\sigma}}\tau\right)d\tau$$

instantaneous frequency:

$$\omega(t) = \frac{t}{\sigma}$$

resolution $\propto$

$$\sigma = \frac{\partial^2\phi(\omega)}{\partial\omega^2}$$

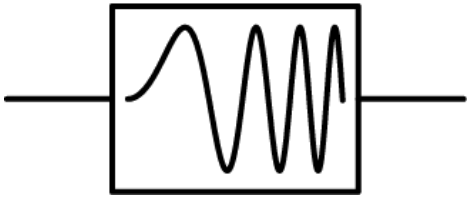
rampless:

$$\frac{T_{\text{in}}^2}{2\pi|\sigma|} \ll 1$$

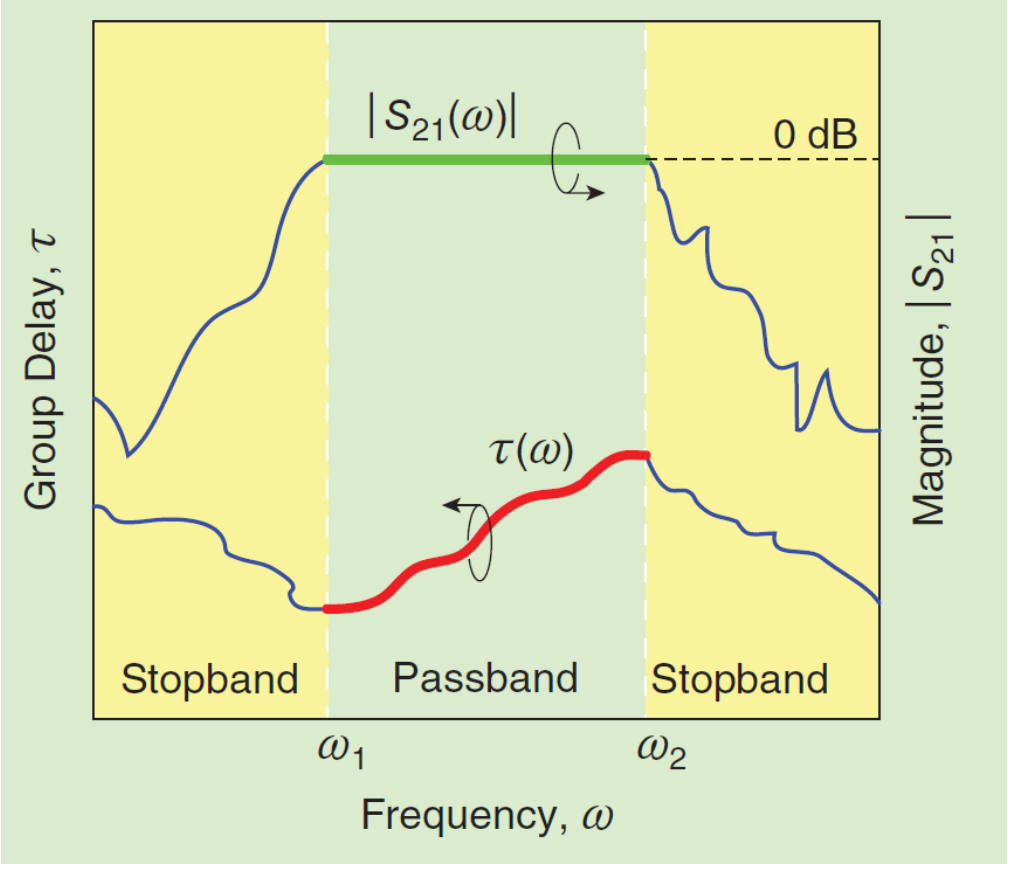
Real-time processing:

- spectrum analyzers
- spectrum sniffers
- radar
- spectroscopy

Ideal Phaser



Arbitrary Group Delay
&
Flat Magnitude
in operation band

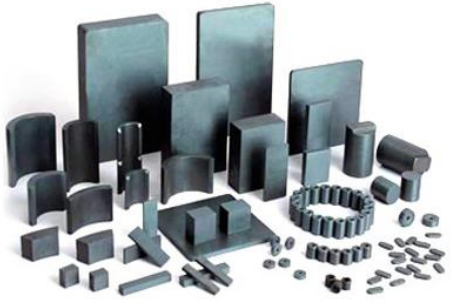


Two Roads for Phasers

Material Dispersion

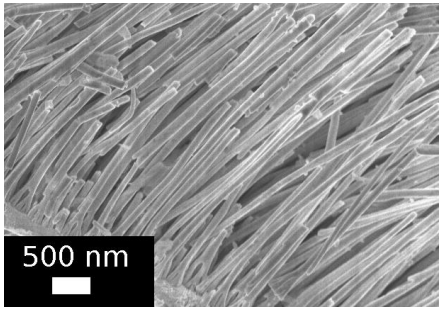
$$\mathbf{D} = \bar{\bar{\varepsilon}}(\omega) \cdot \mathbf{E} + \bar{\bar{\xi}}(\omega) \cdot \mathbf{H}$$
$$\mathbf{B} = \bar{\bar{\zeta}}(\omega) \cdot \mathbf{E} + \bar{\bar{\mu}}(\omega) \cdot \mathbf{H}$$

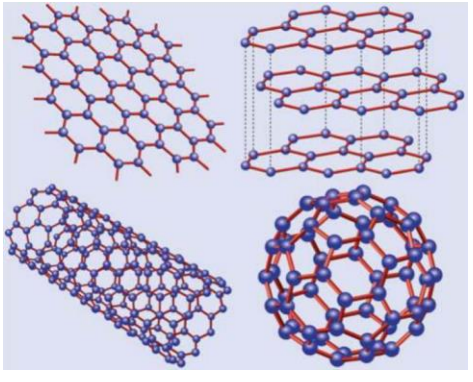
Ferromagnetic Materials



Metamaterials

Nano-structured Materials





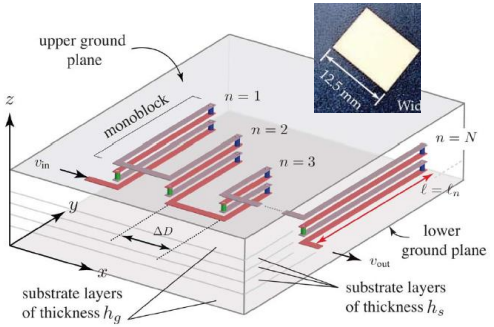
Best for radiative devices

Network Dispersion

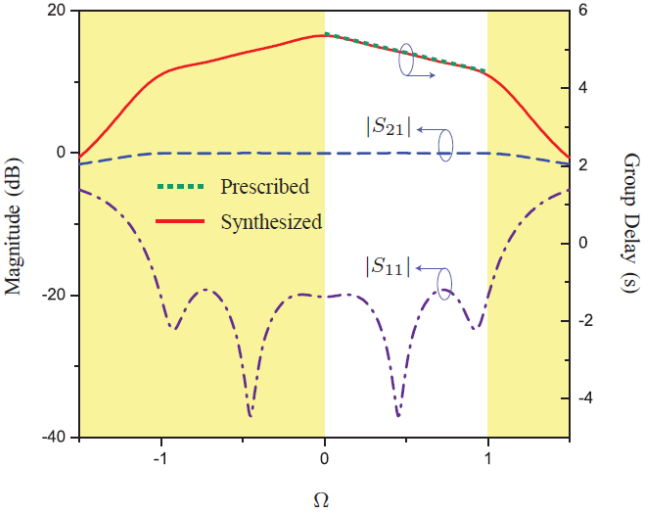
$$S_{21}(\omega) = |S_{21}(\omega)|e^{j\Phi(\omega)}$$

$$\tau(\omega) = \frac{d\Phi(\omega)}{d\omega}$$

Equalizers



Equalized Filters

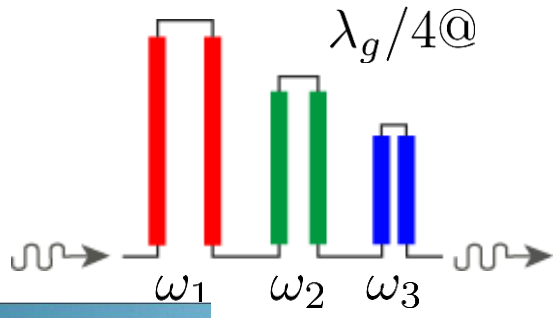


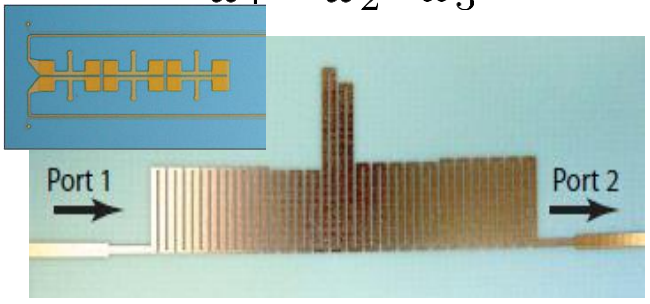
Mathematical Prototype

Best for guided-wave devices

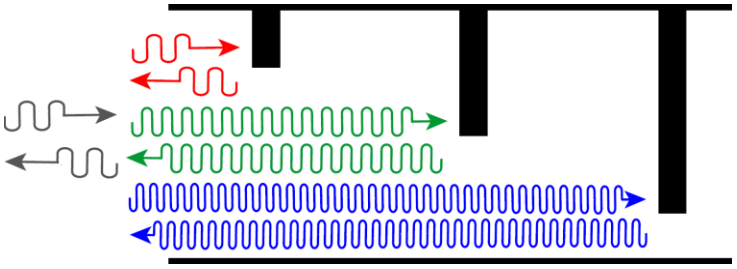
Dispersive Network Phasers

C-Section Transmission (All-Pass)

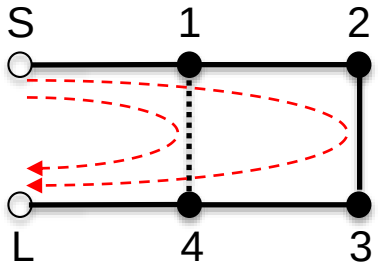


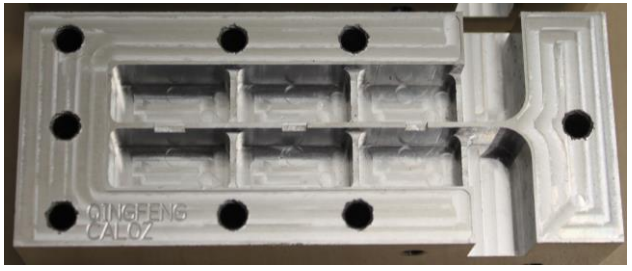


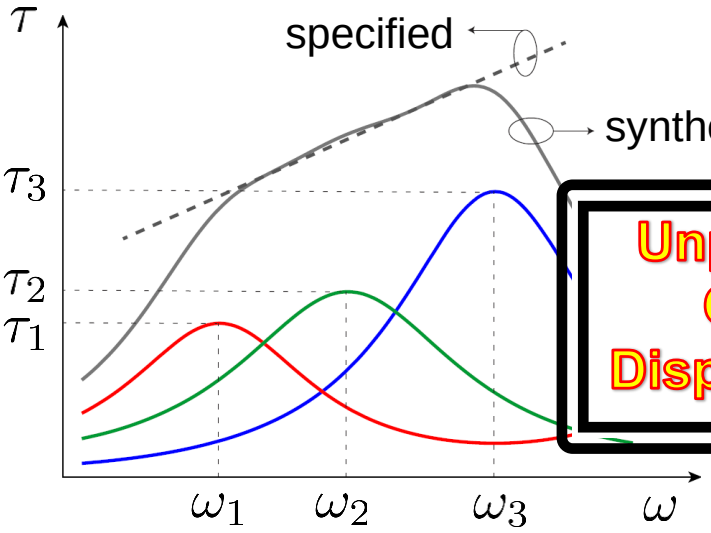
Direct-Coupled Reflection (All-Pass)



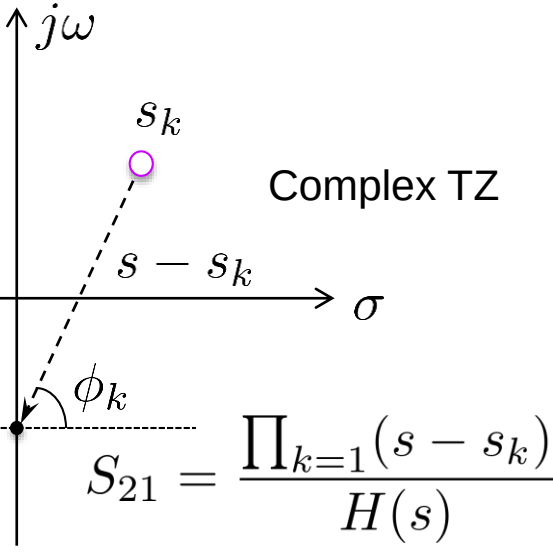
Cross-Coupled Transmission (Band-Pass)







Unprecedented Control of Dispersive Phase

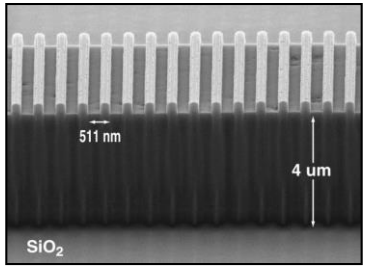


C. Caloz, S. Gupta, Q. Zhang, and B. Nikfal, *IEEE Microw. Mag.*, vol. 14, no. 6, pp. 87-103, Sept. 2013.

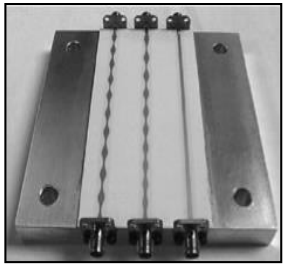
"Conventional" Microwave Phasers

Reflection-type (1-port) Phasers

Optical Bragg Gratings (microwave photonics)

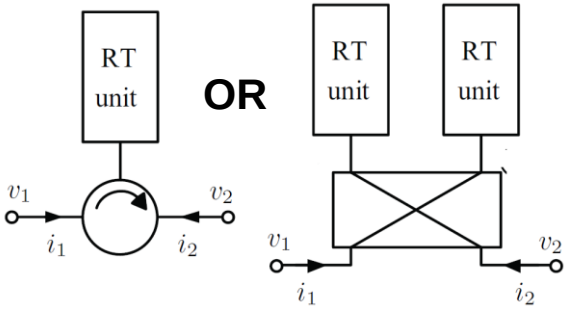


Chirped Lines



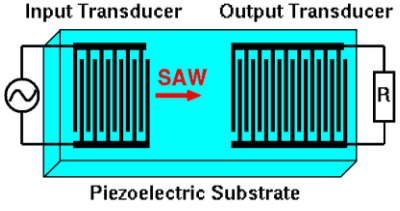
Modified All-Pass Filters

2-port transformation



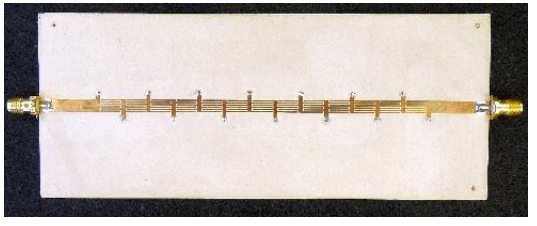
Transmission-type (2-port) Phasers

Surface Acoustic Wave Devices

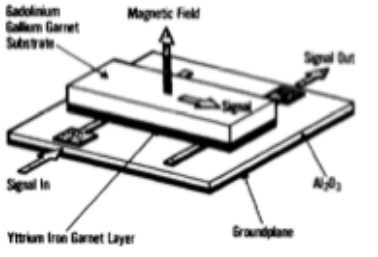


Cross-coupled Structures

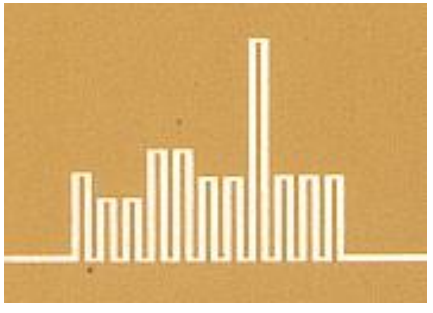
Composite Right/Left-Handed (CRLH)TLs

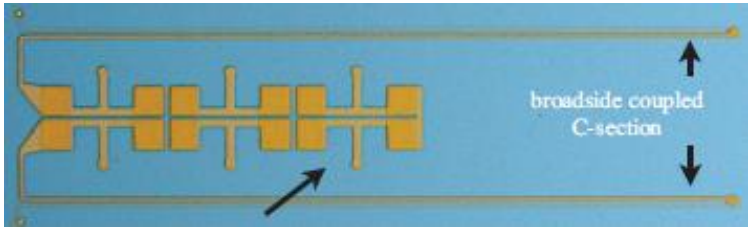


Magnetostatic Wave Devices



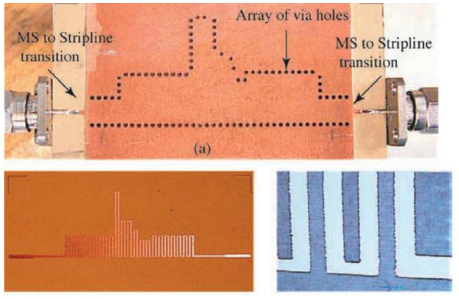
C-section based all-pass networks



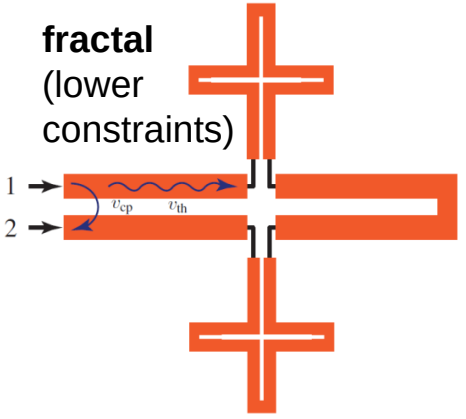


Recent Phasers: Higher Performance & Agility

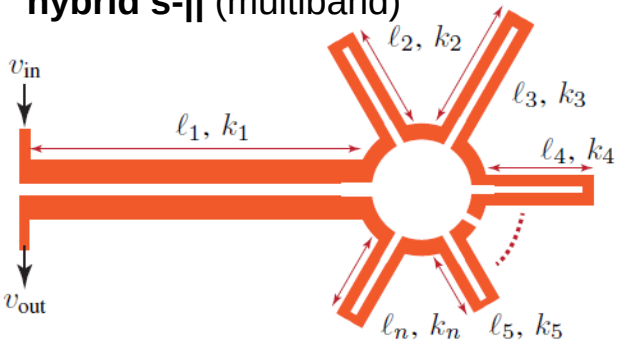
packaged (non-radiative)



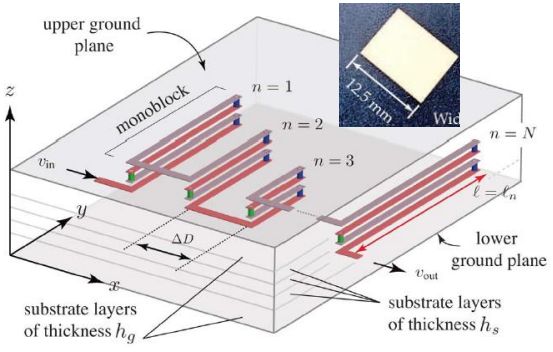
fractal (lower constraints)

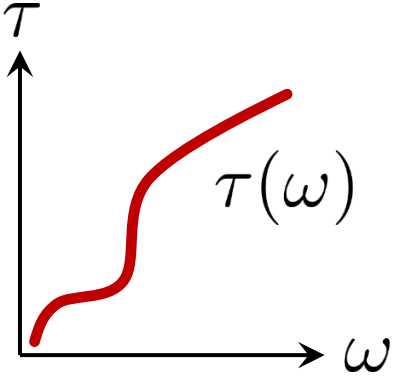


hybrid s-|| (multiband)

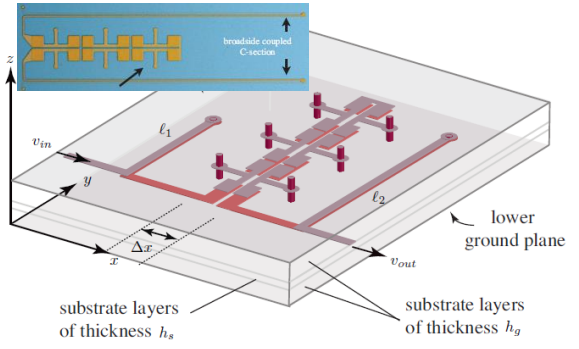


multilayer (compact)



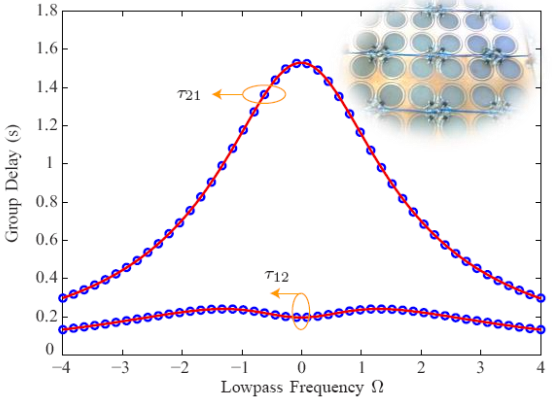


Multilayer CRLH (enhanced BW)



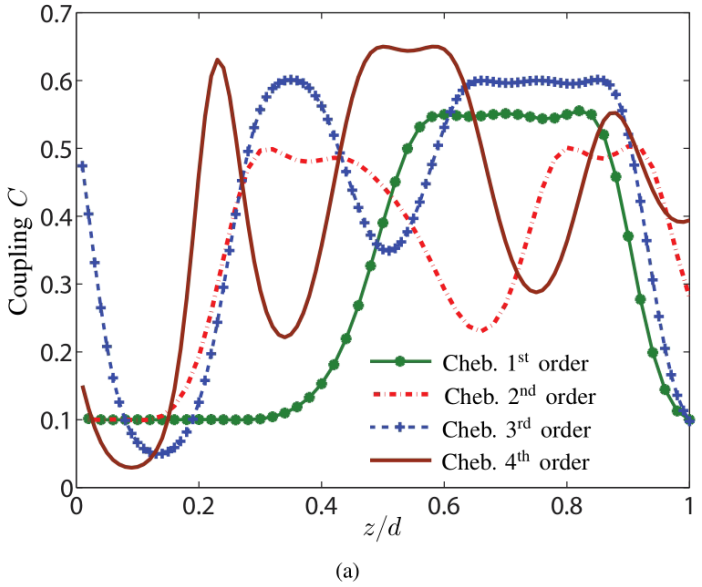
x-coupled (+magnitude)

nonreciprocal (relays)

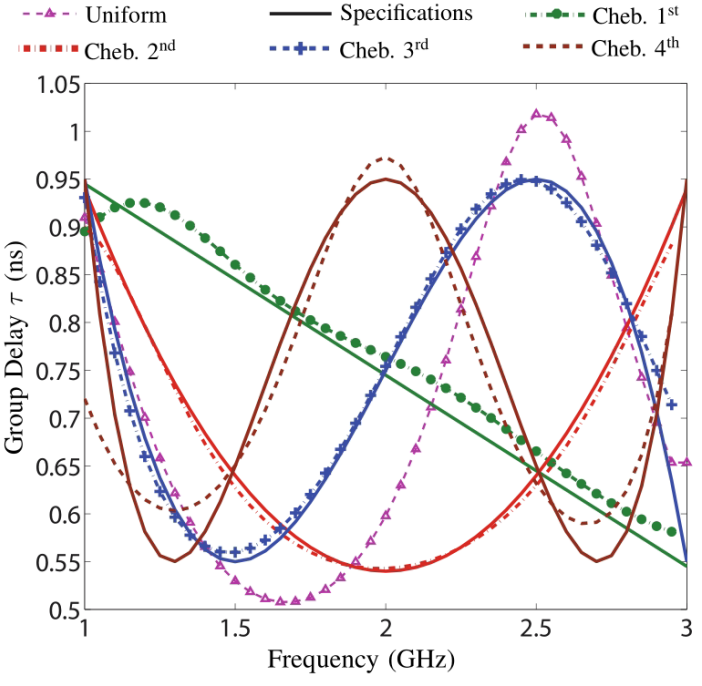


Nonuniform (Chebyshev)

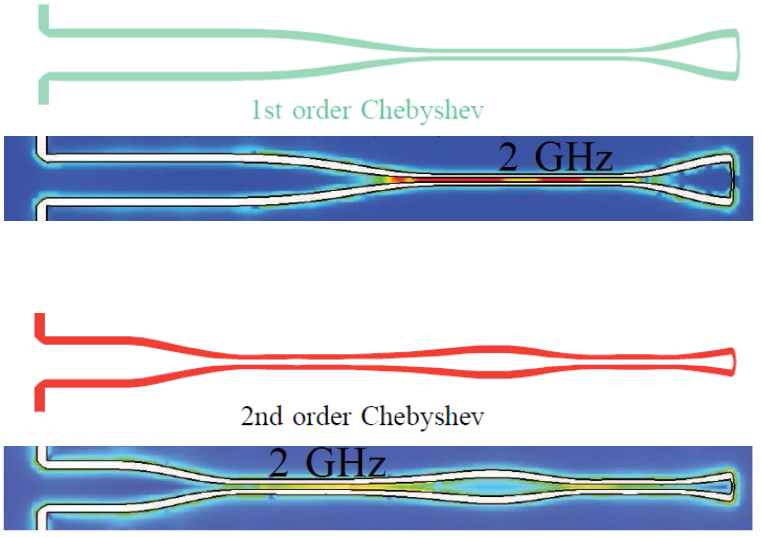
Nonuniform Phaser with Chebyshev Response

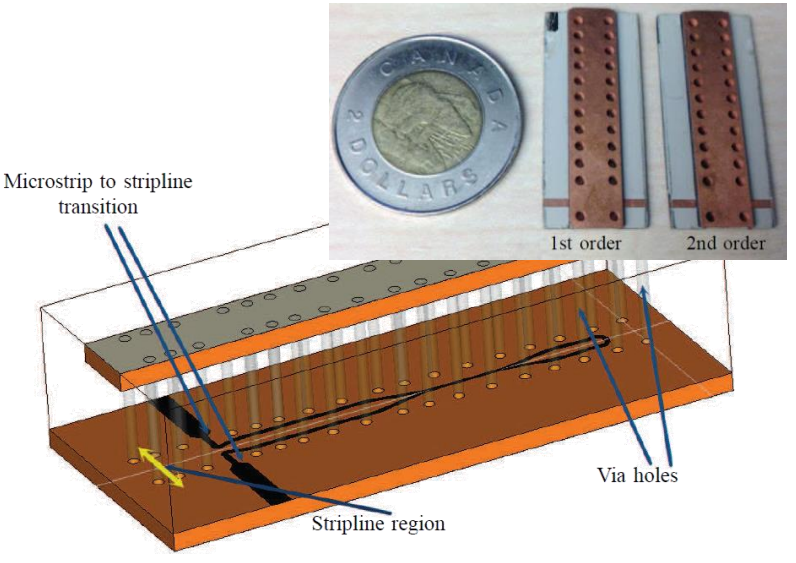


(a)



(b)

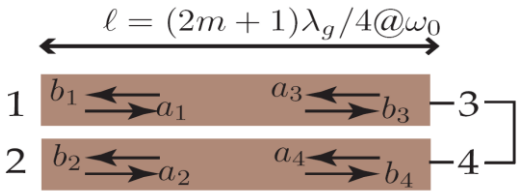


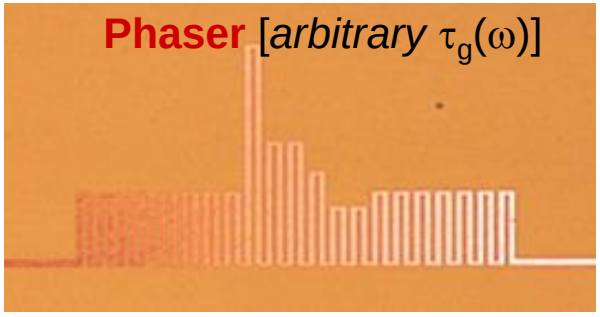


S. Taravati, S. Gupta, Q. Zhang, and C. Caloz, arXiv, Dec. 2014.

Metamaterial Dispersion Boosting

Structure : Micro-Scale

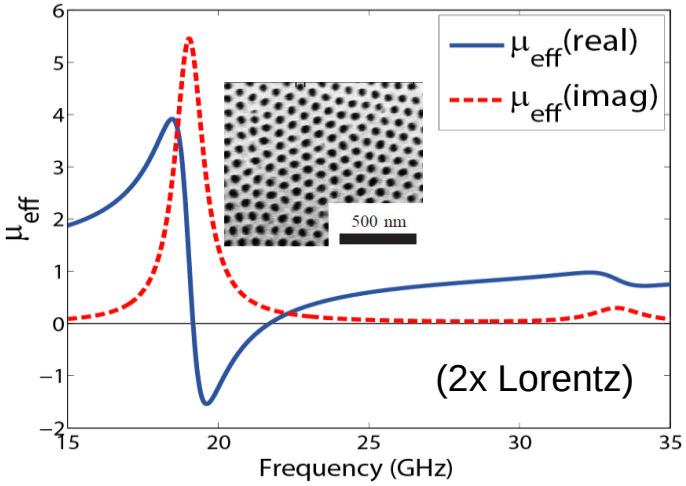




$$\tau_g(\theta) = \frac{2a}{a^2 + (1-a^2)\cos^2\theta}\frac{d\theta}{d\omega} \qquad \left(a = \sqrt{\frac{1-k}{1+k}}\right)$$

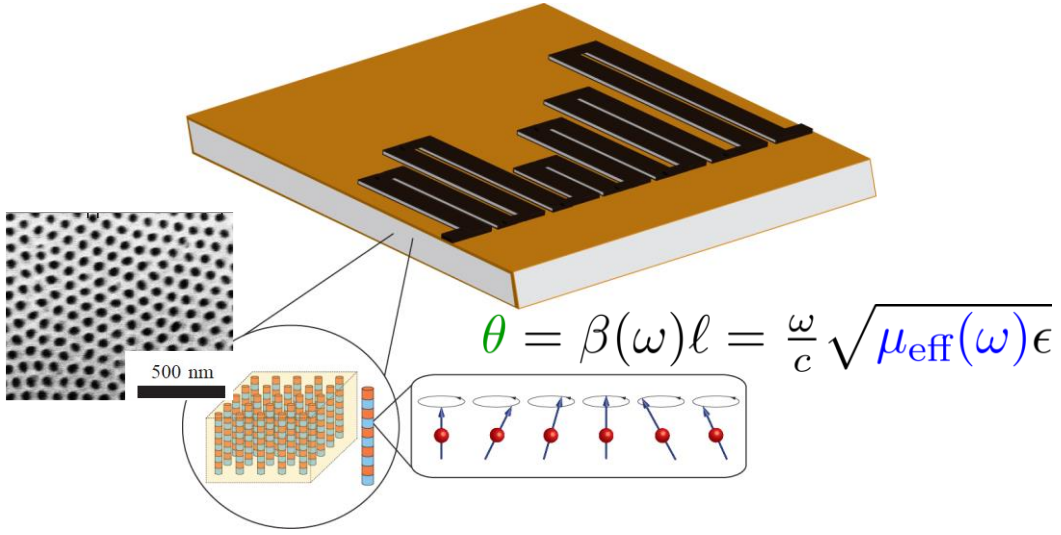
S. Gupta, A. Parsa, E. Perret, R. V. Snyder, R. J. Wenzel and C. Caloz, *IEEE T-MTT*, vol. 58, no. 9, pp. 2392-2407, Sept. 2010.

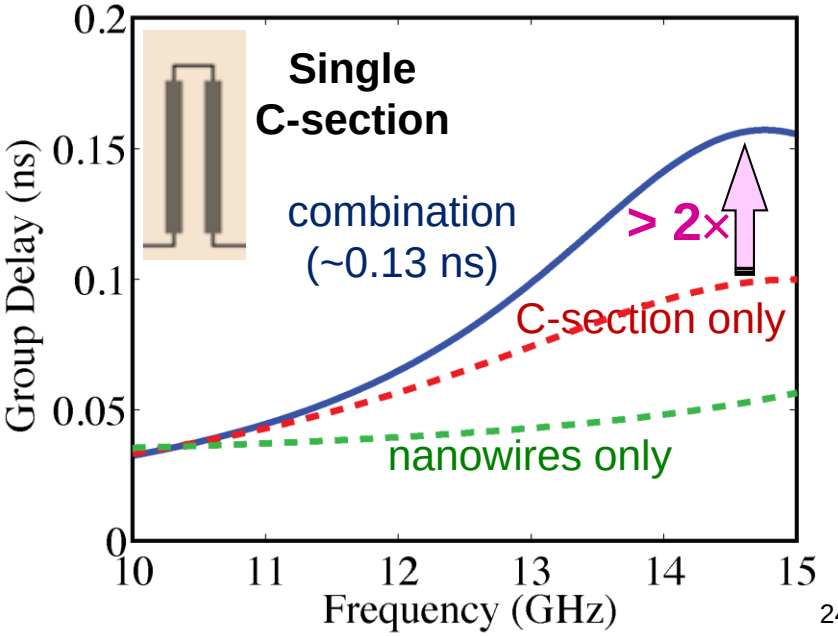
Material : Nano/Atomic-Scale



L.-P. Carignan, A. Yelon, D. Ménard, and C. Caloz, *IEEE T-MTT*, vol. 59, no. 19, pp. 2568–2586, Oct. 2011.

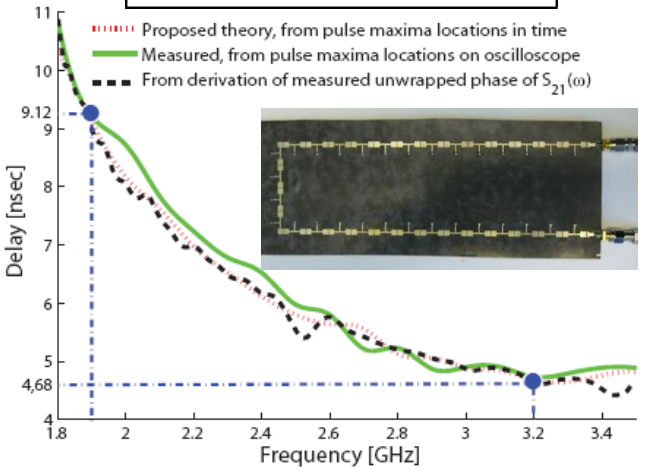
Tri-Scale MTM for Processing Resolution Enhancement

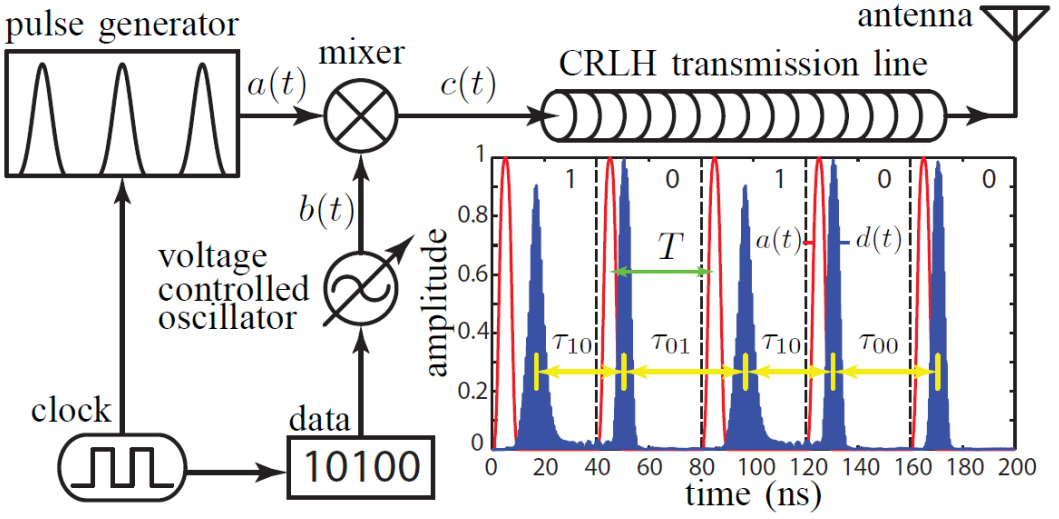


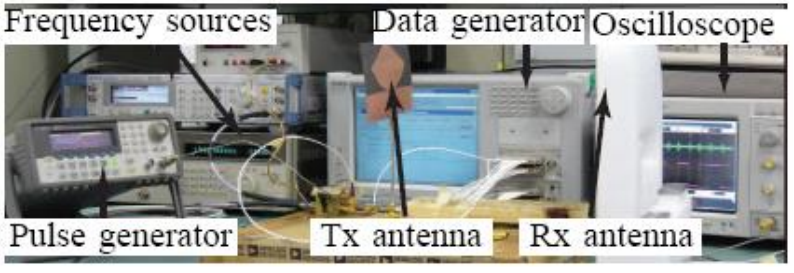


Pulse Position Modulator

$$\tau_{g,\mathrm{CRLH}} = f(\omega_c)$$







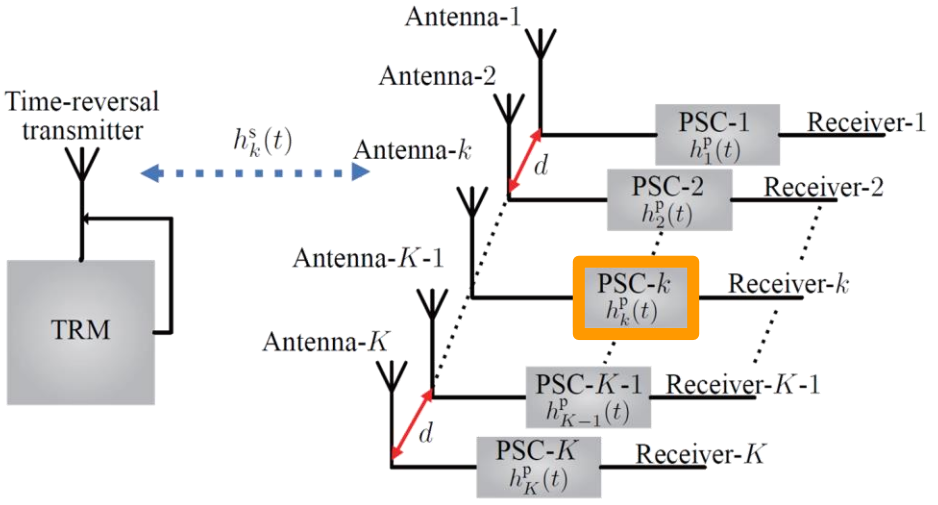
Modulated signal

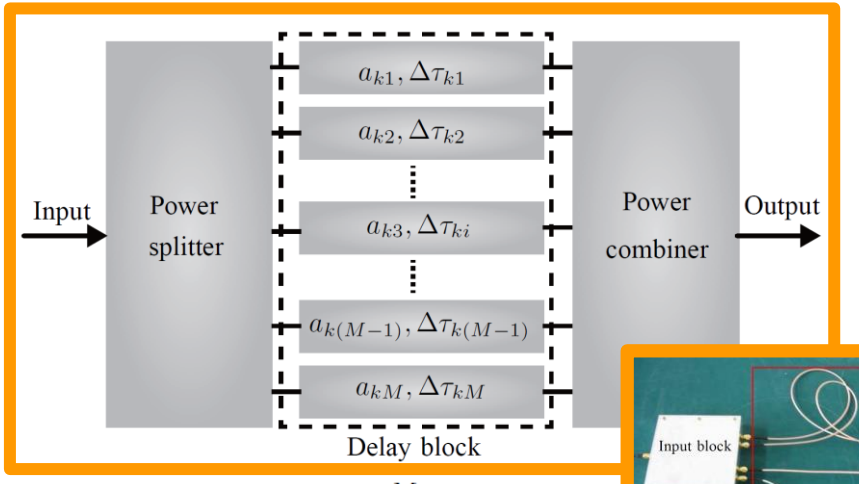
$$d(t,\omega_c) = B\cos(\omega_c t)\sum_{n=0}^{\infty} p\left[t - nT_f - \tau_g(\omega_c)\right]$$

- ☺ **simplicity**
- ☺ **non-coherent Tx/Rx**
- ☺ **ω-scalable**

H. V. Nguyen, and C. Caloz, *IEEE MWCL*, vol. 18, no. 6, pp. 527-529, Aug. 2008.

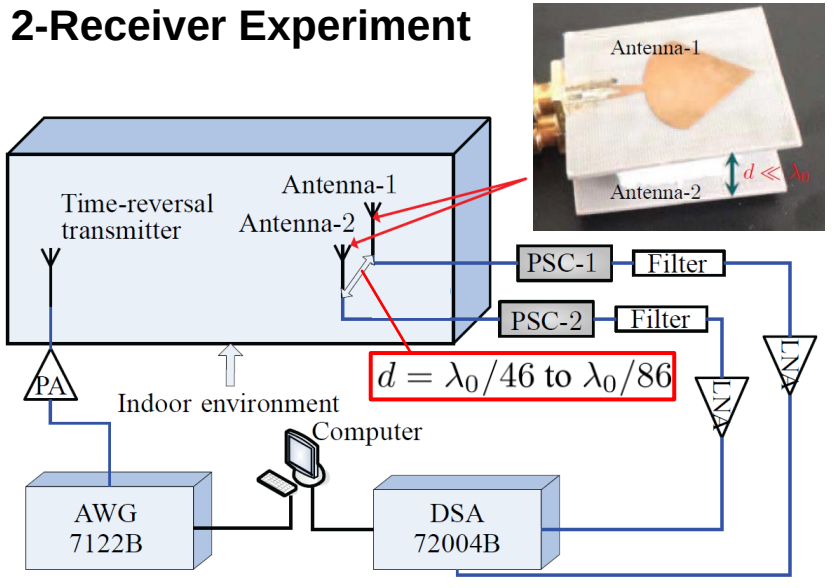
PSC-Dispersion-Enhanced Sub-λ Time Reversal

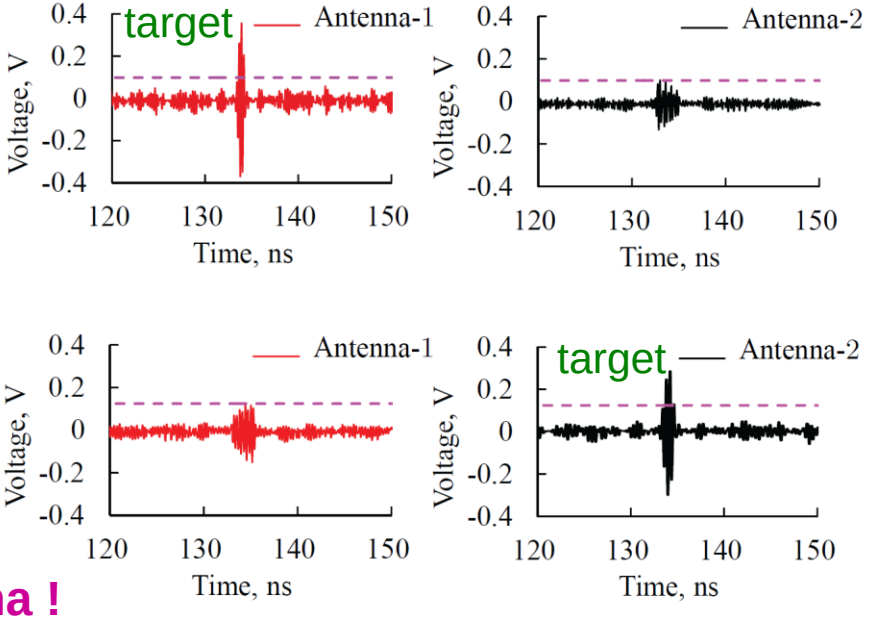




PSC Phaser: $H_k^p(\omega) = \sum_{i=1}^{M} a_{ki} e^{-j\omega\Delta\tau_{ki}}$

2-Receiver Experiment

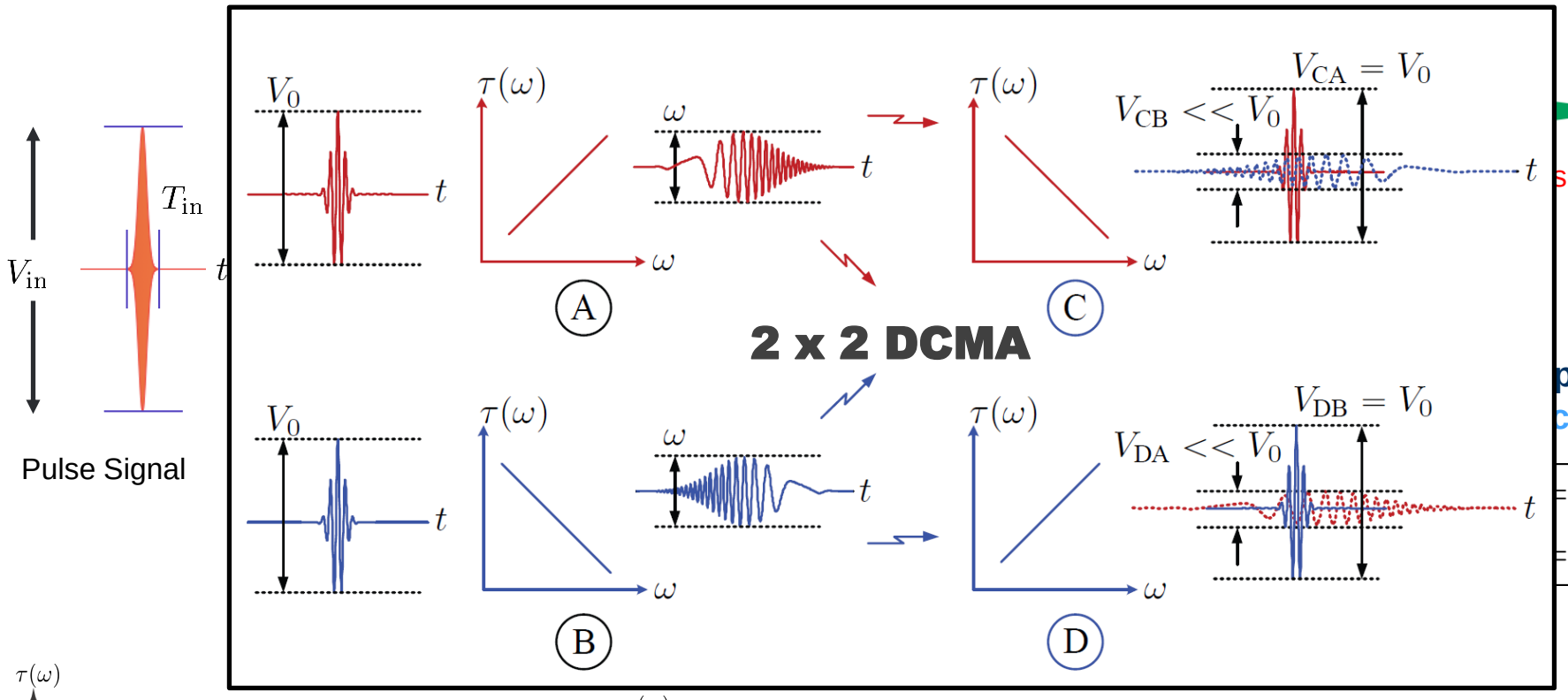


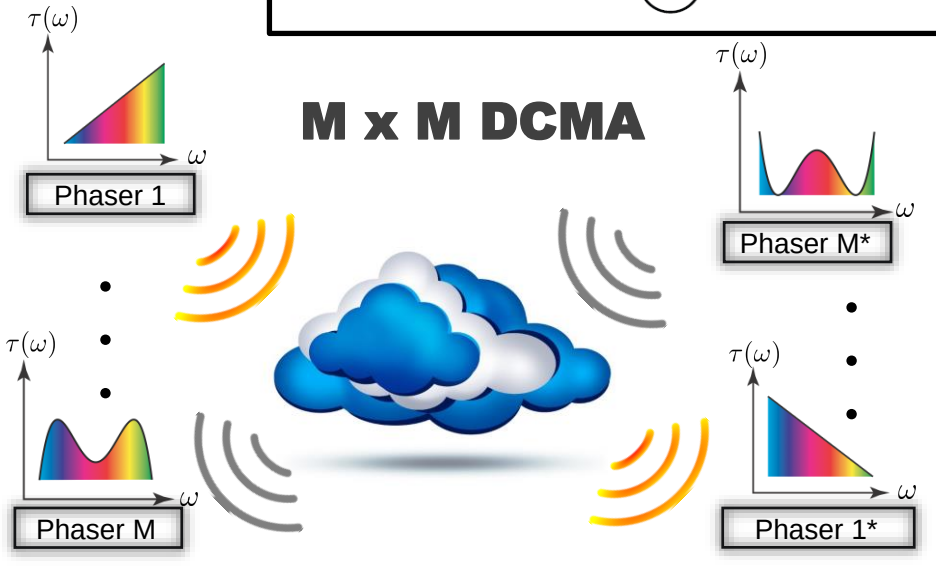


No need for big TR Mirror, simple Tx antenna !

S. Ding, R. Zang, L. Zou, B. Wang, and C. Caloz, arXiv:1407.7085, Aug. 2014.

Dispersion Code Multiple Access (DCMA)

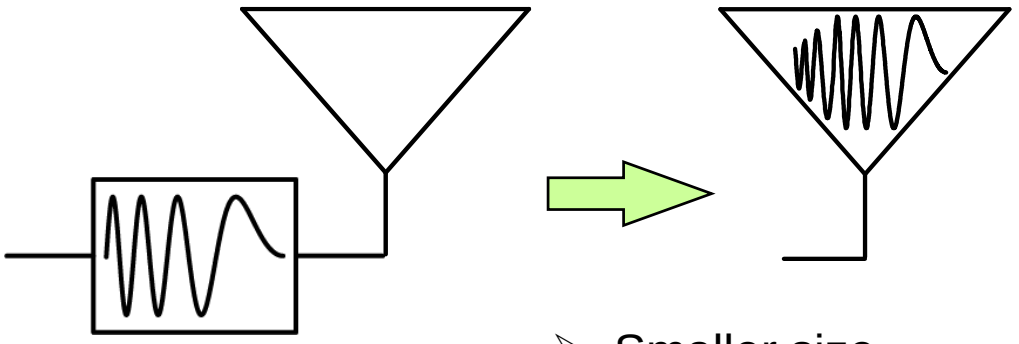




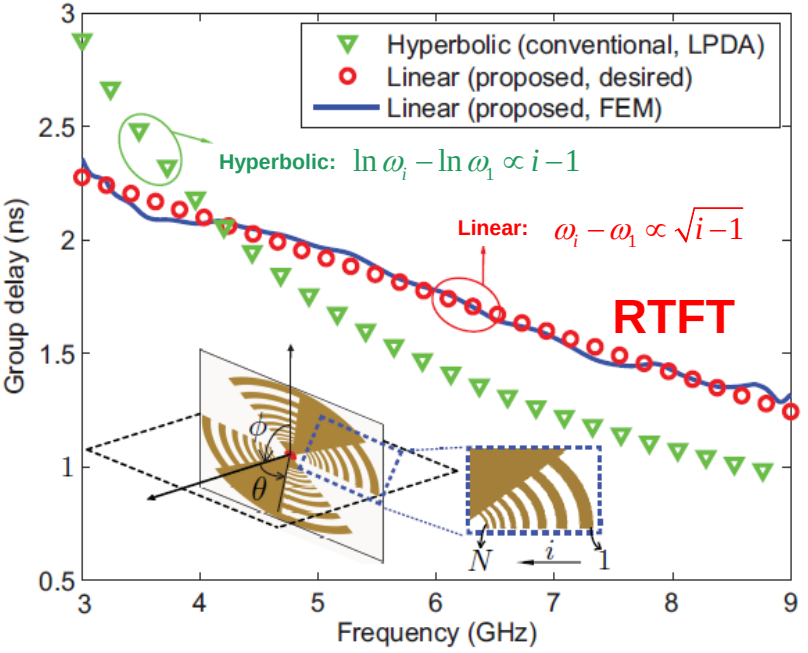
- **Analog & real-time ⇒ low-cost, broadband, frequency scalable & ultra fast**
- **Robust to multipath fading**
- **Robust to noise**
- **Reconfigurable via Time-Reversal BS**

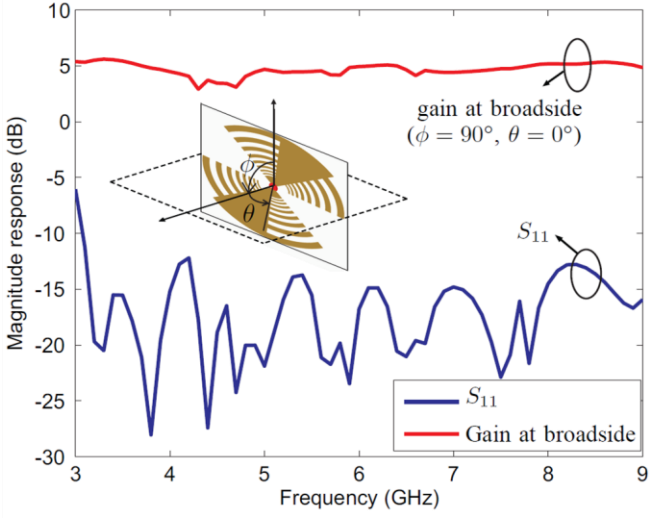
Pending Patent: B. Nikfal, M. Salem, and C. Caloz, *US 62/002,978*, Nov. 2013.

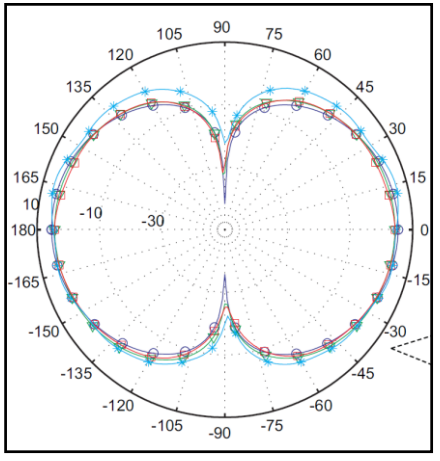
“Phasenna”: Phaser + Antenna

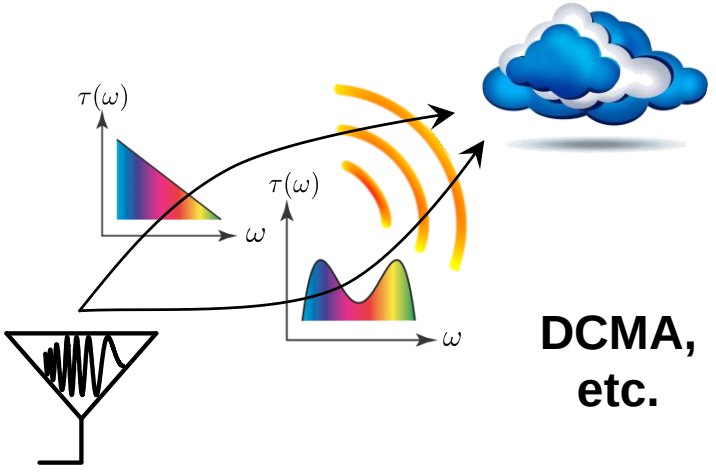


- Smaller size
- Lower loss
- EM processing









DCMA, etc.

L. Zou and C. Caloz, “Concept of combined phaser-antenna – “phasenna”? – for wireless radio analog signal processing (R-ASP),” in IEEE AP-S Int. Antennas Propag. (APS), Memphis, TN, Jul. 2014, pp. 822–823.

The Time DE – Phasers QUIZ

1) **Current technologies can meet the predicted needs of 5G.**

   ABSOLUTLY NOT! 3x order performance increase, little clue…

2) **How does R-ASP compare with DSP & OSP.**

   Same flexibility as DSP

   Same speed as OSP

3) **A phaser is a directed-energy weapon in Star Trek.**

   YES, BUT ALSO also the core of R-ASP

4) **Phasers are the same thing as filters.**

   NO, phase design rather than magnitude design

5) **What is the most fundamental operation of a phaser?**

   Real-time Fourier analysis

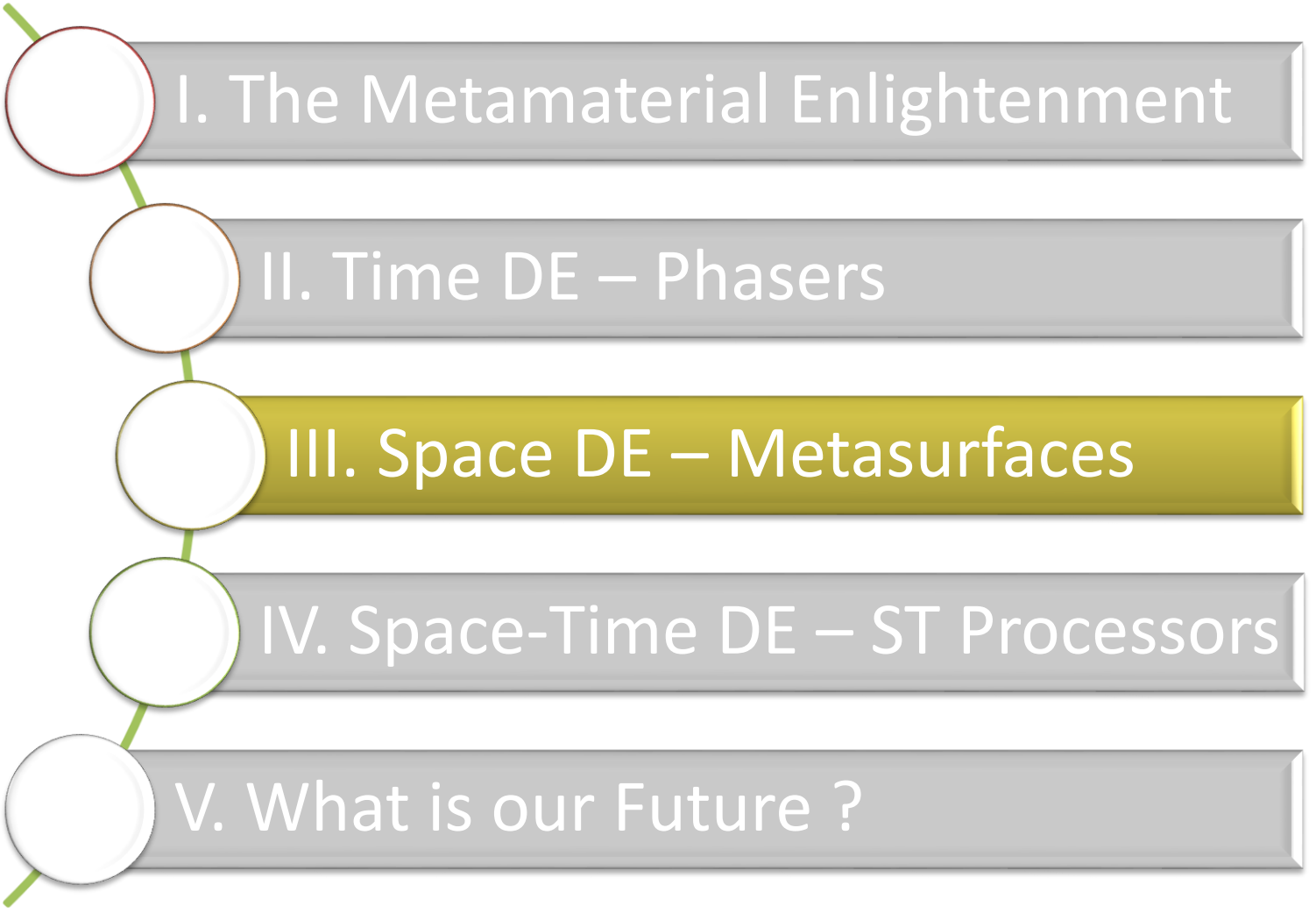
6) **Decisive progress have been made DE in the past lustrum.**

   YES, components & synthesis

7) **What is the connection between MTMs and phasers?**

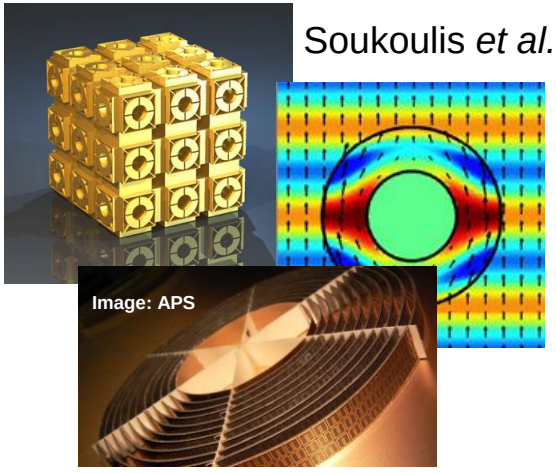
   MTMs are inherently dispersive, phasers are engineered as dispersive.

Outline


I. The Metamaterial Enlightenment
II. Time DE – Phasers
III. Space DE – Metasurfaces
IV. Space-Time DE – ST Processors
V. What is our Future ?

Spatial Counterpart of Phasers: Metasurfaces

3D Metamaterials



Soukoulis *et al.*

Smith *et al.*

☺ Extra Field Control

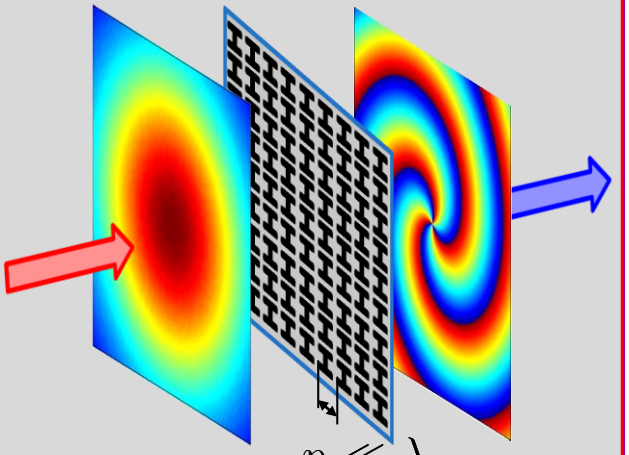
☺ Signal Processing

☹ Bulkiness

☹ High Loss

☹ Fabrication Difficulty

Metasurfaces



$p \ll \lambda_0$

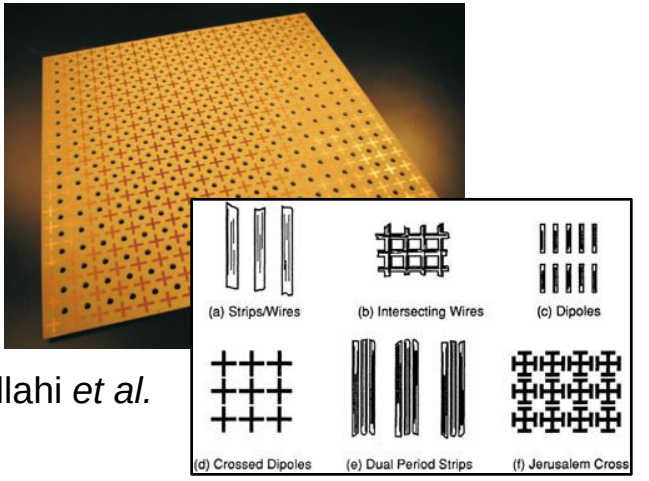
☺ Extra Field Control

☺ Extensive Processing

☺ Reduced Dimensionality

- ☺ Small Profile
- ☺ Low Loss
- ☺ Fabrication Easiness

Frequency Selective Surfaces (FSSs)



Fallahi *et al.*

☺ Reduced Dimensionality

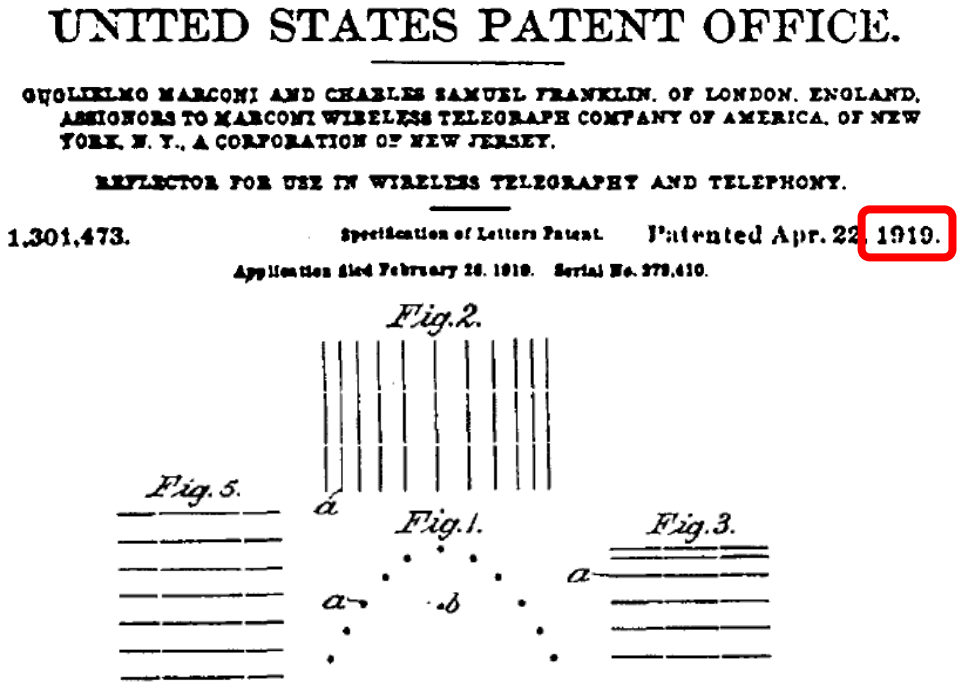
☺ Filtering & Polarization

☹ Limited Signal Processing

☹ Uniformity Restriction

Dimensional Reduction of Metamaterials & Functional Extension of FSSs

Frequency-Selective Surfaces (FSSs)



UNITED STATES PATENT OFFICE.

GUGLIELMO MARCONI AND CHARLES SAMUEL FRANKLIN, OF LONDON, ENGLAND, ASSIGNORS TO MARCONI WIRELESS TELEGRAPH COMPANY OF AMERICA, OF NEW YORK, N. Y., A CORPORATION OF NEW JERSEY.

REFLECTOR FOR USE IN WIRELESS TELEGRAPHY AND TELEPHONY.

1,301,473. Specification of Letters Patent. Patented Apr. 22, 1919.

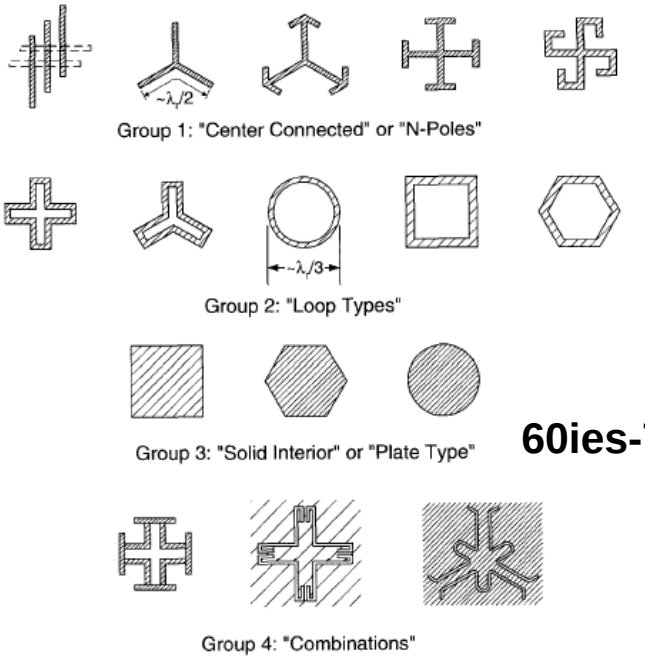
Application filed February 26, 1919. Serial No. 279,410.

Fig. 2.

Fig. 5.

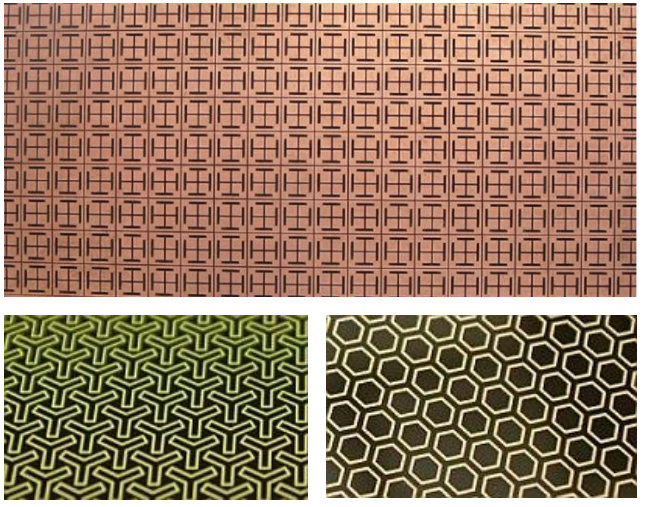
Fig. 1.

Fig. 3.



60ies-70ies

- Temporal frequency filtering
- Uniformly periodic structures
- Resonant elements
- Circuit model synthesis

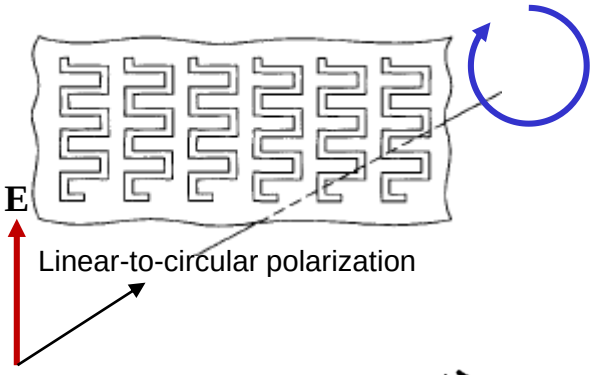


G. Marconi and C. S. Franklin, Reflector for use in telegraphy and telephony, Application filed Feb. 1919.

B. A. Munk, Frequency Selective Surfaces: Theory and Design, Wiley-Interscience, 1 Ed, 2000.

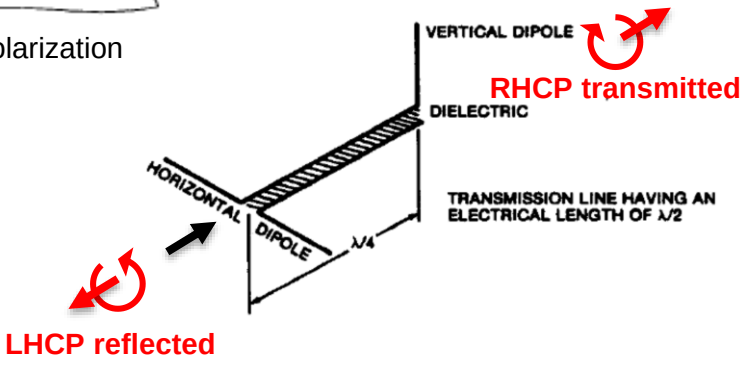
Circular Polarization Selective Surface (CPSS)

- **CP polarizers**

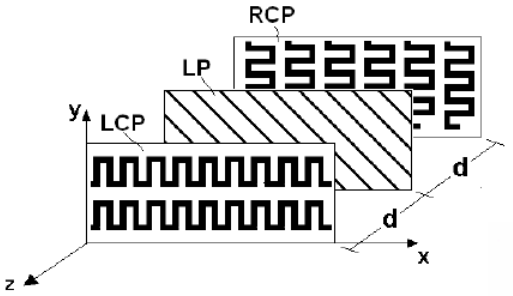


- **CPSS: reflects one CP and transmits other:**

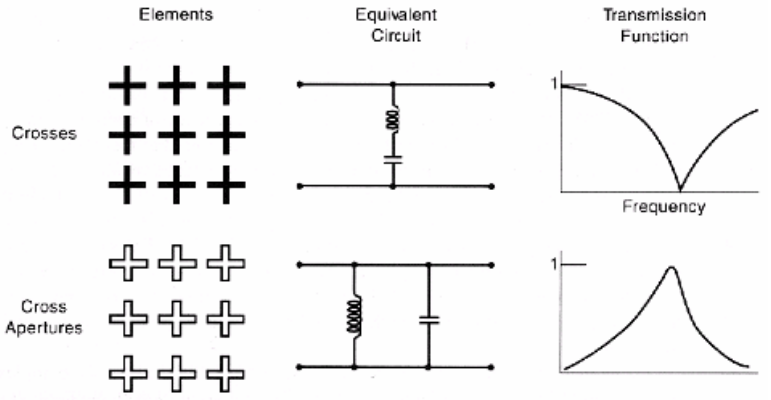
  first CPSS → ⊥ ½-λ dipoles[1]



- **Cascaded CPSSs for enhanced bandwidth & performance**



- **Circuit model synthesis** →

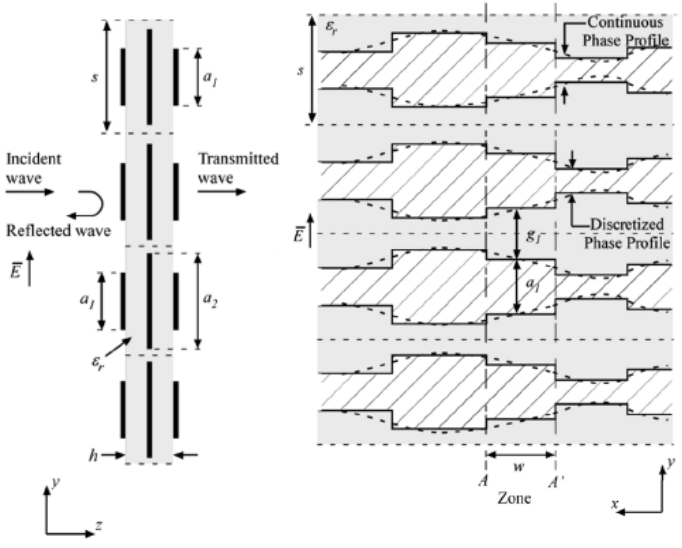


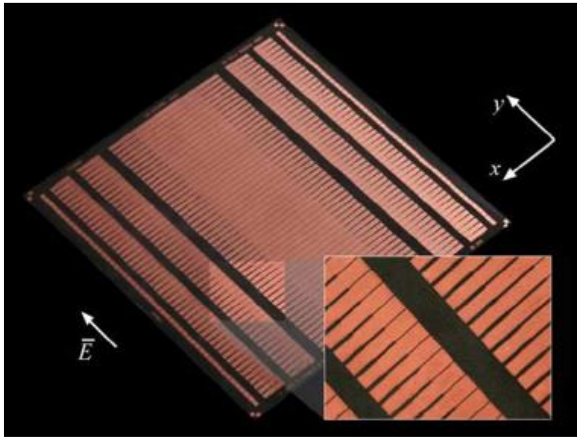
[1]W. V. Tilston, T. Tralman, and S. M. Khanna, "A polarization selective surface for circular polarization," *Antennas and Propagation Society International Symp., 1988. AP-S. Digest* , vol., no., pp.762-765 vol.2, 6-10 June 1988.

Reflect Arrays & Transmit Arrays

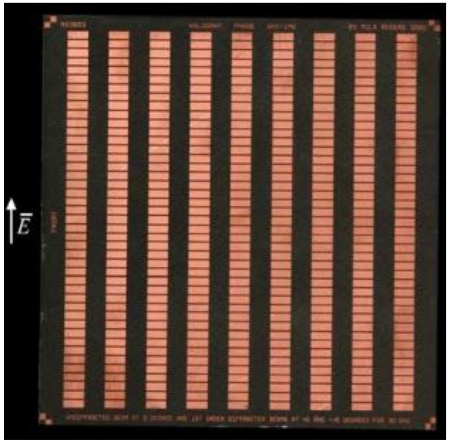
- flexible amplitude & phase control
- Non-periodic for polarization control and beam-focussing
- E.g.: Phase-shifting surfaces (PSS) for wavefront shaping and lensing[1]
- Full-wave analysis synthesis

Phase-Shifting Surfaces (PSS)





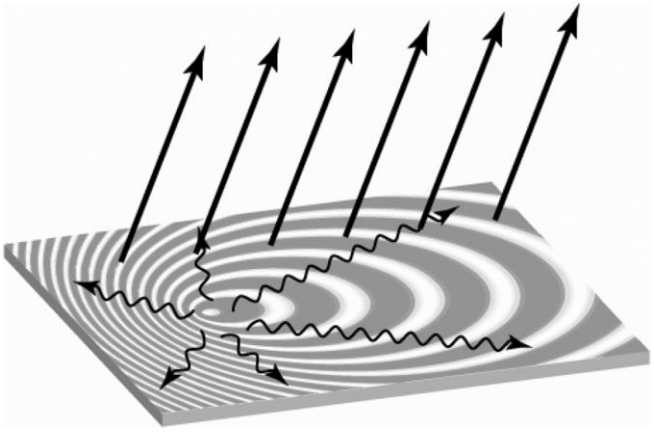
Cylindrical Lens

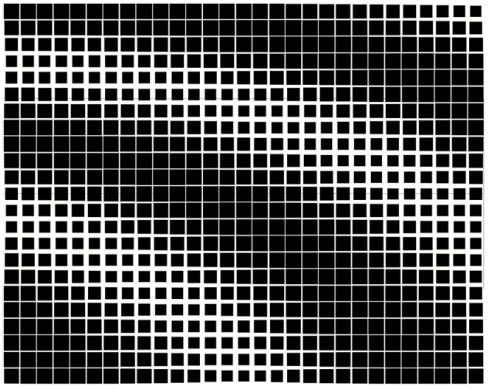


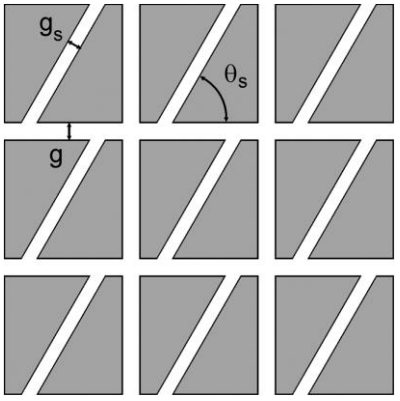
Binary phase plate

[1]N.Gagnon, A. Petosa, and D. A. McNamara, "Thin Microwave Quasi-Transparent Phase-Shifting Surface (PSS)," *Antennas and Propagation, IEEE Transactions on* , vol.58, no.4, pp.1193,1201, April 2010.

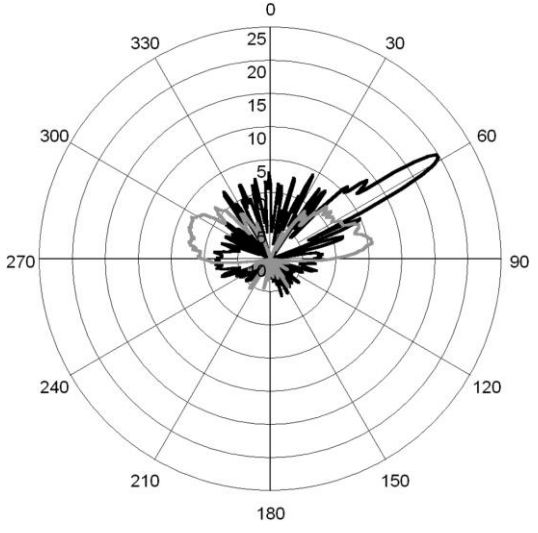
Leaky-Wave Holographic Beam Forming

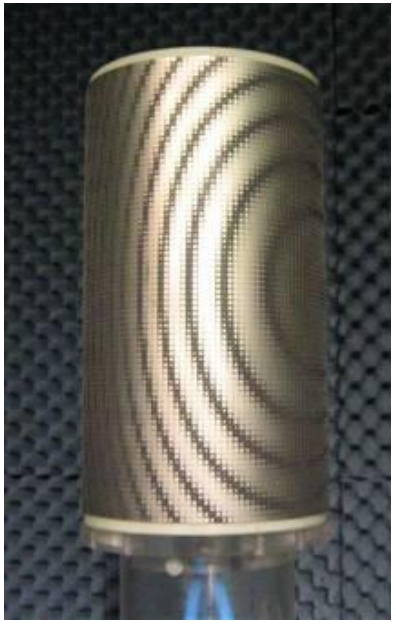


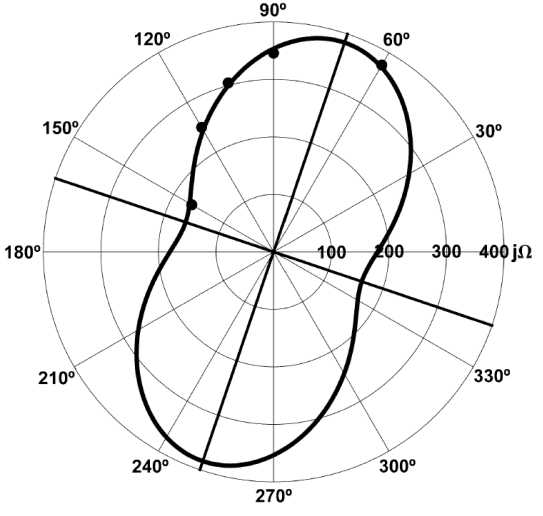




$$Z(\mathbf{x}_t) = j\left[X + M\mathrm{Re}\left(\psi_{\mathrm{rad}}\psi^*_{\mathrm{source}}\right)\right]$$

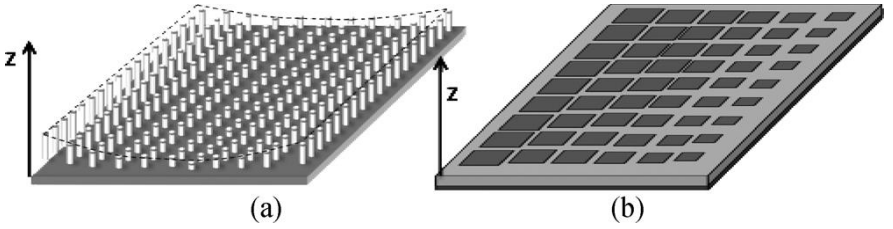


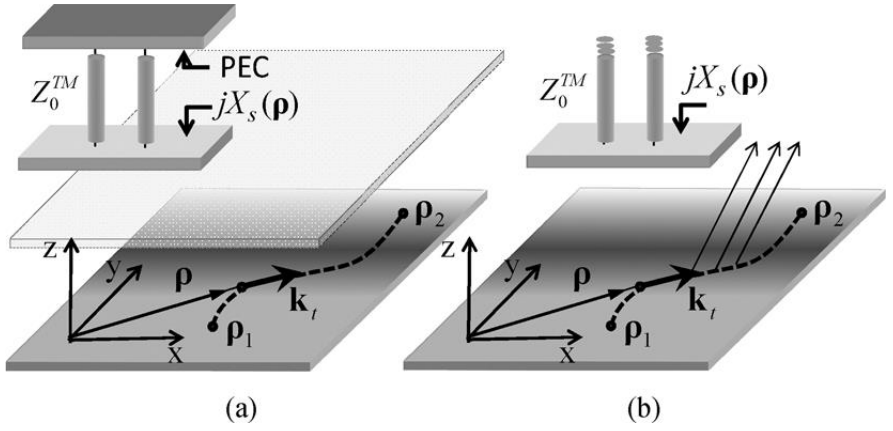




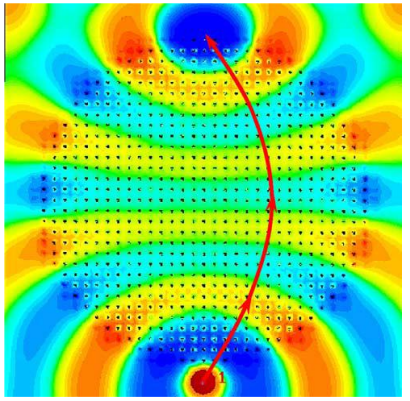
B. H. Fong et al., IEEE Trans. Antennas Propagat., 58, 2010.

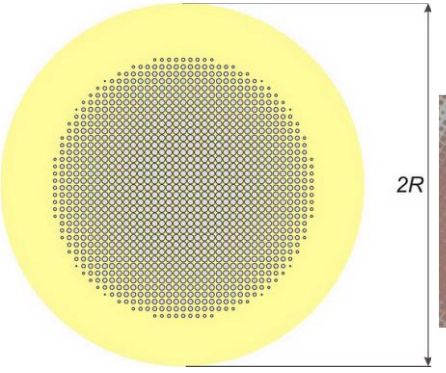
Surface/Leaky-Wave Phase/Beam Forming

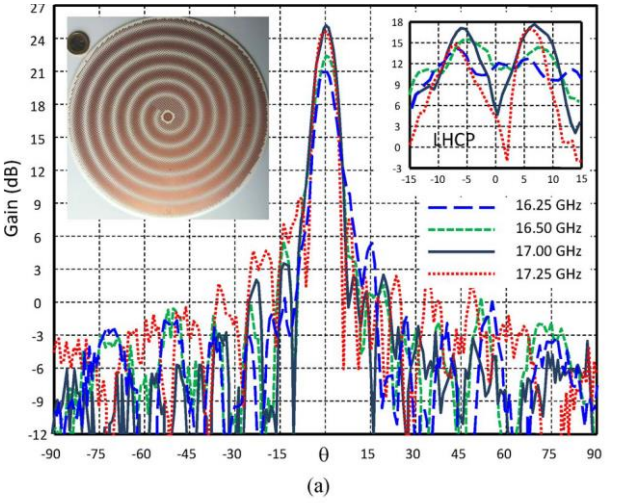


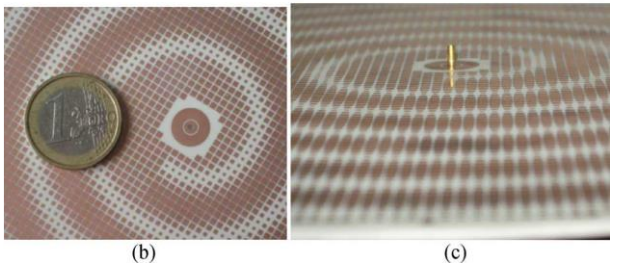


$$X_s(w, k_t) = \frac{\zeta k_{z1} \tan(h' k_{z1})}{k\epsilon_r - k(\epsilon_r + 1)\delta k_{z1} \tan(h' k_{z1})}$$







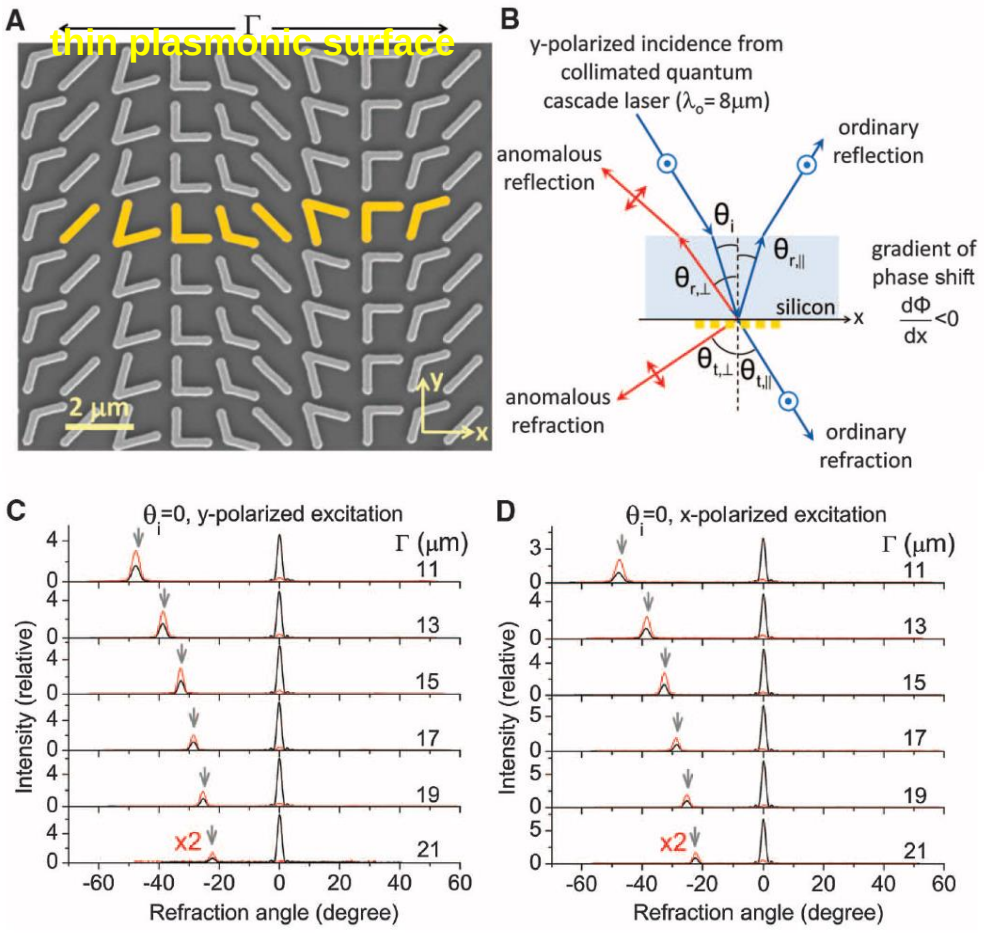


S. Maci et al., IEEE Antennas Wirel. Propagat. Lett., 10, 2011.

Generalized Metasurfaces

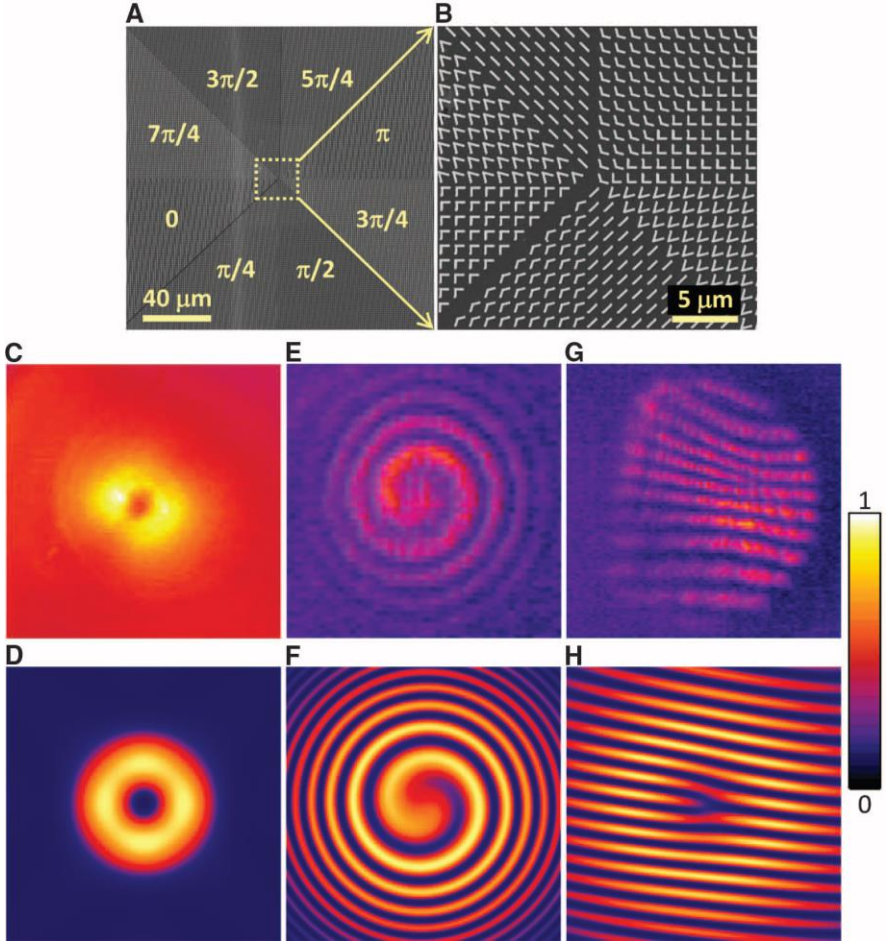
Generalized Refraction

$$n_t \sin\theta_t - n_i \sin\theta_i = \frac{\lambda_0}{2\pi}\frac{d\Phi}{dx}$$



Vortex wave

$$\Psi \propto \exp(in\phi)$$

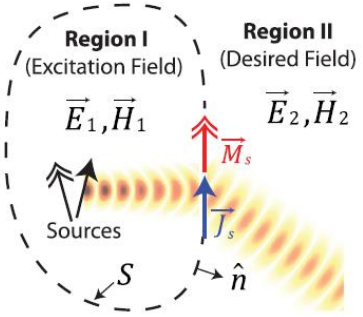


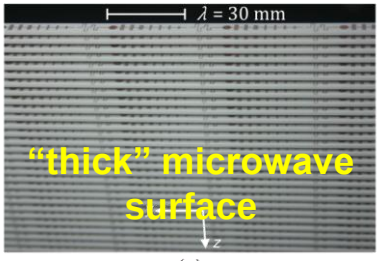
N. Yu et al., Science, 334, 2011.

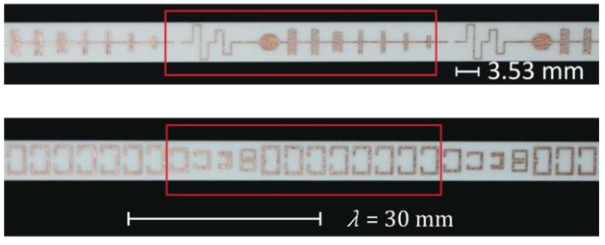
“Huygens” Surfaces

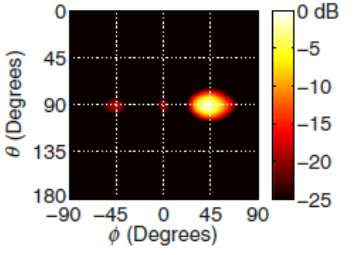
Impedance Based Synthesis

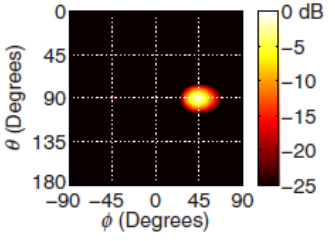
$$\mathbf{J}_s = \hat{\mathbf{n}} \times (\mathbf{H}_2 - \mathbf{H}_1) \rightarrow \bar{\bar{Y}}_{es}$$
$$\mathbf{M}_s = -\hat{\mathbf{n}} \times (\mathbf{E}_2 - \mathbf{E}_1) \rightarrow \bar{\bar{Z}}_{ms}$$



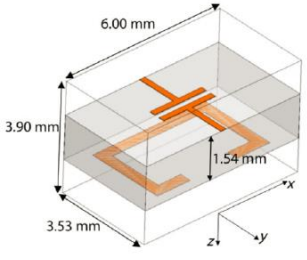


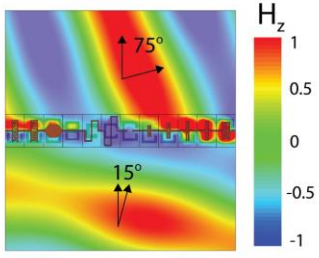


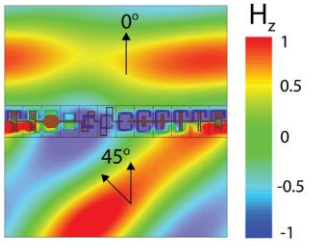


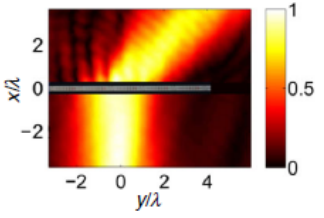


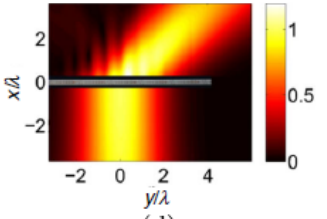
arbitrary refraction



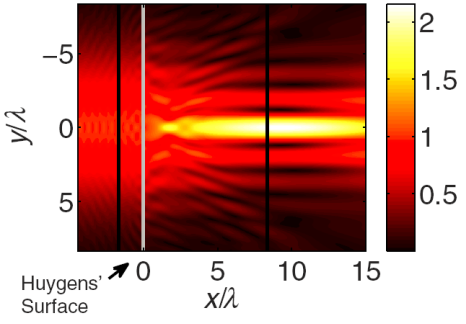


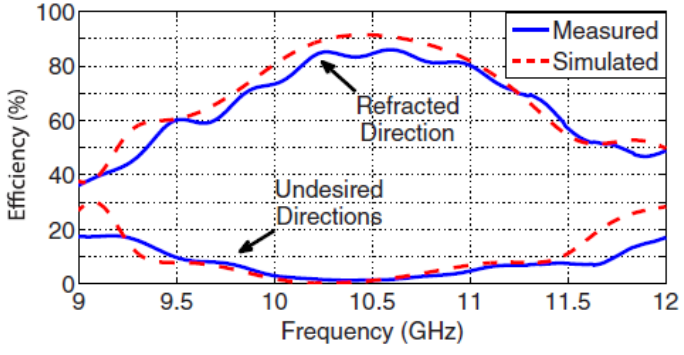






“Bessel” beam

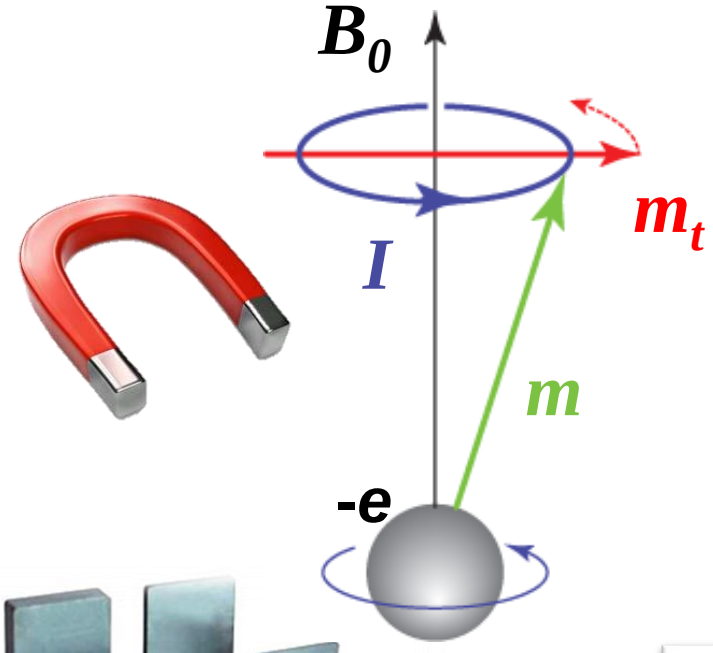


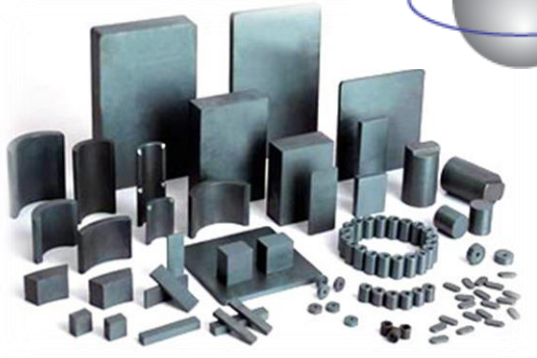


C. Pfeiffer and A. Grbic, Phys. Rev. Lett., 110, 2013.

Magnetless Magnetism

Magnetic Materials

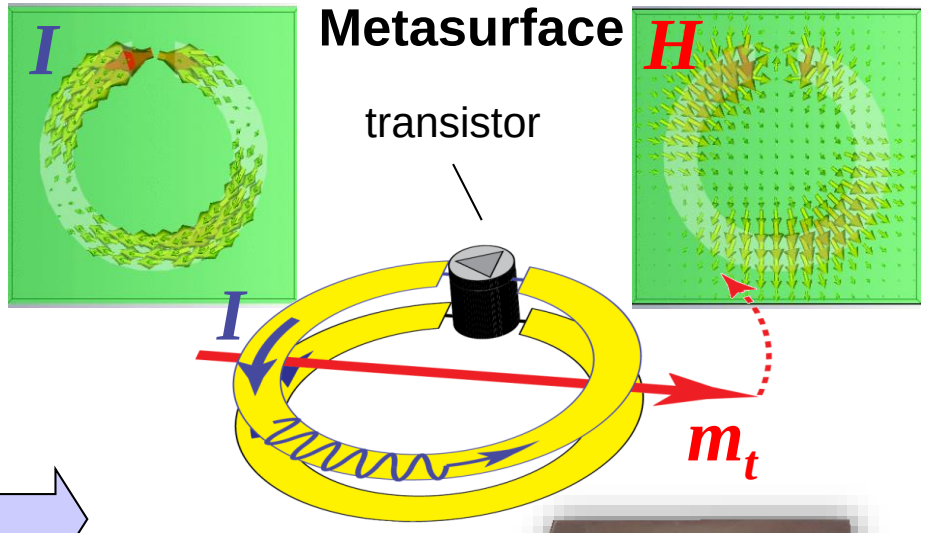




electron spin precession

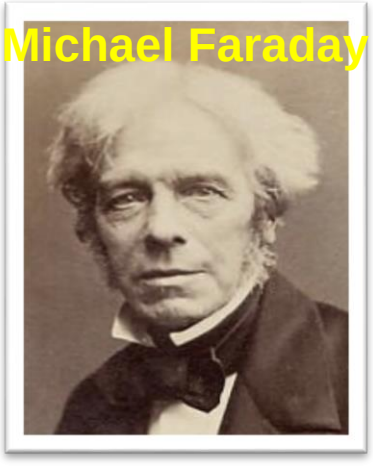
$$\omega_0 = \gamma B_0$$

Magnetless Metasurface



traveling-wave ring

$$\omega_{0n} = \frac{nc}{R\sqrt{\epsilon_{\text{eff}}}}$$

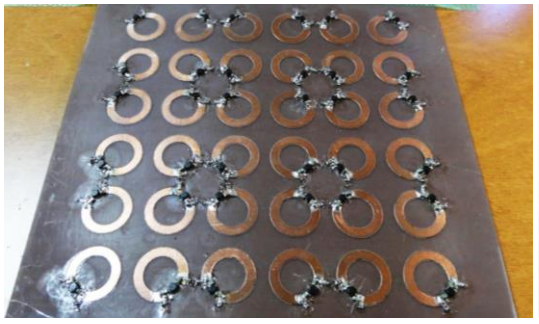


T. Kodera, D. L. Sounas, and C. Caloz, *Appl. Phys. Lett.*, vol. 99, p. 031114, July 2011.

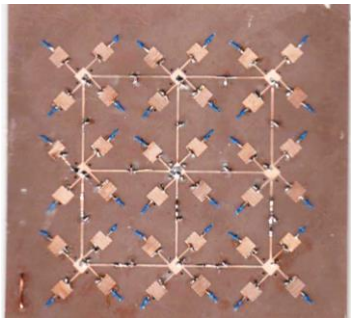
Faraday Reflection Experiment

Prototype

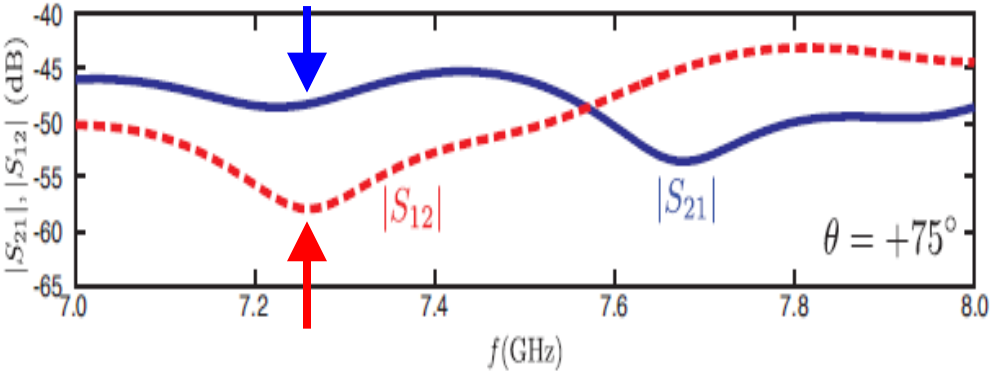
Top

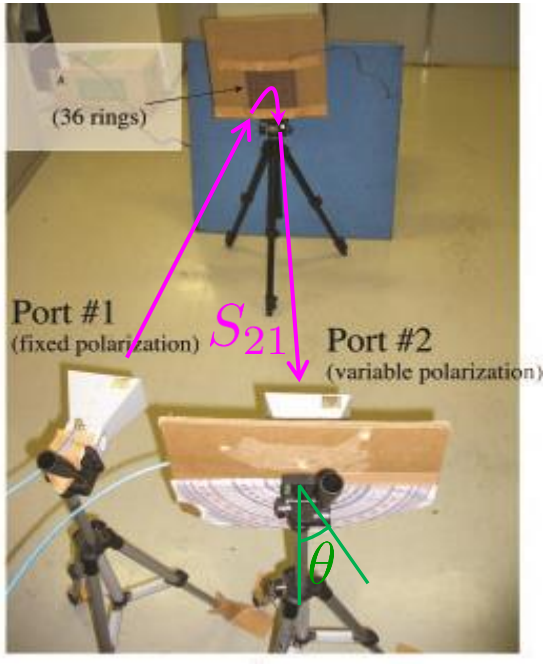


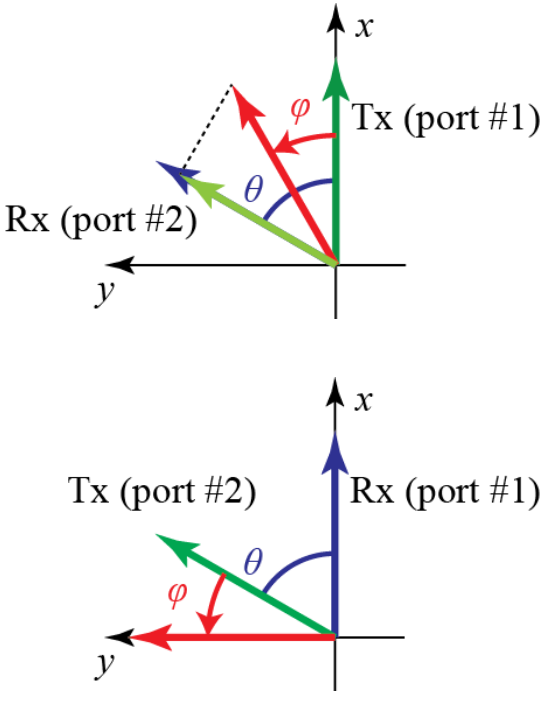
Bottom



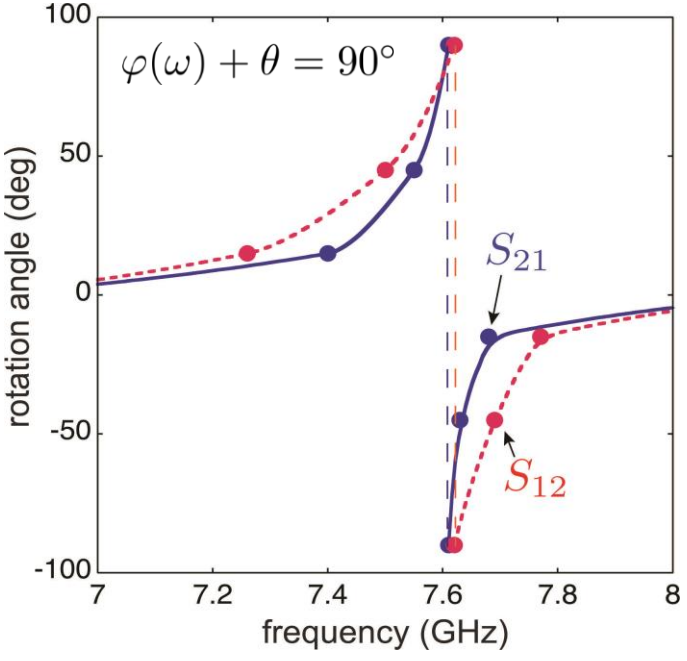
Measured S-parameters







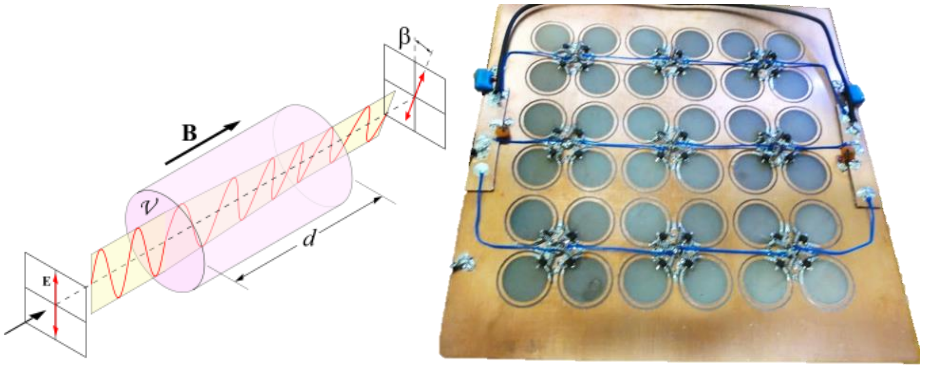
Faraday Rotation



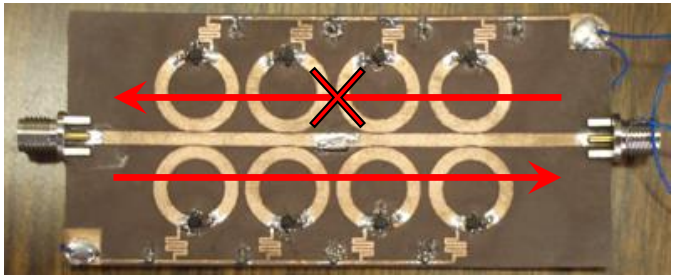
T. Kodera, D. L. Sounas, and C. Caloz, *Appl. Phys. Lett.*, 99, 231902, 2011.

First Applications of I-MNMs

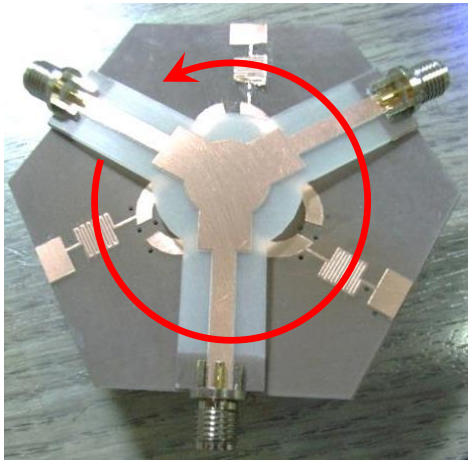
Flat Faraday Rotator



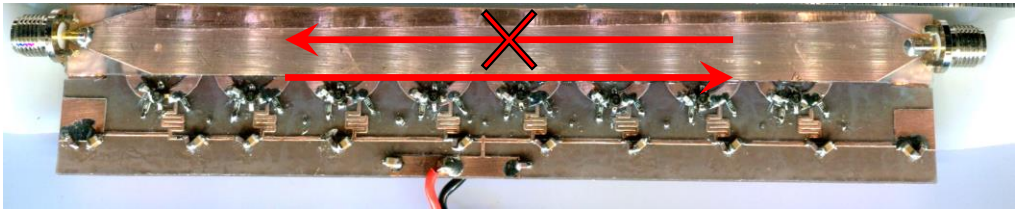
Field Displacement / Resonance Isolators

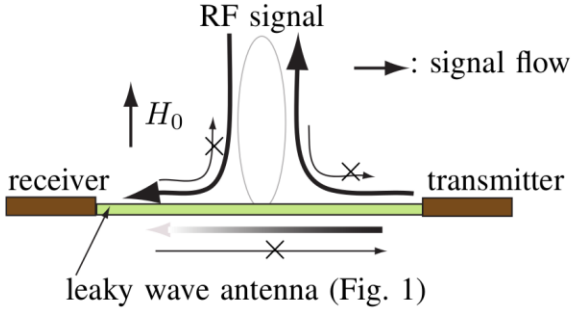


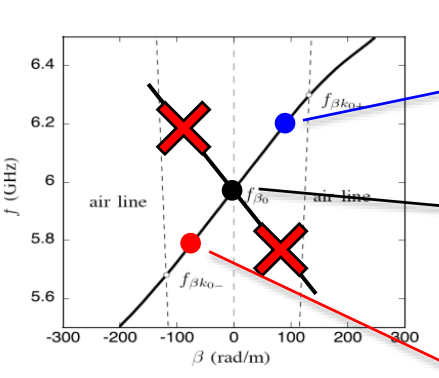
Circulator

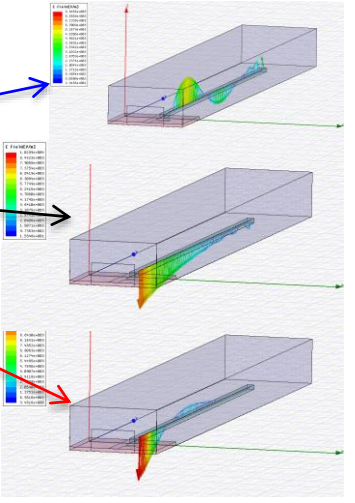


Leaky-Wave Antenna Diplexer





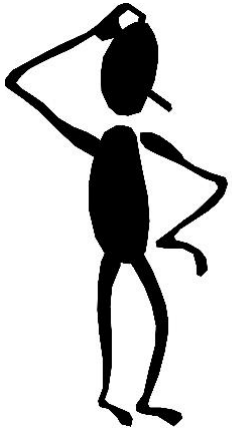




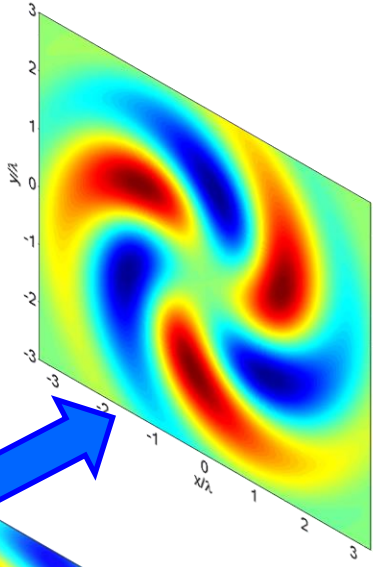
T. Kodera, D. L. Sounas, and C. Caloz, *IEEE T-MTT*, vol. 61, no. 3, pp. 1030-1042, 2013.

T. Kodera, D. L. Sounas, and C. Caloz, *IEEE AWPL*, vol. 10, pp. 1551-1554, 2011.

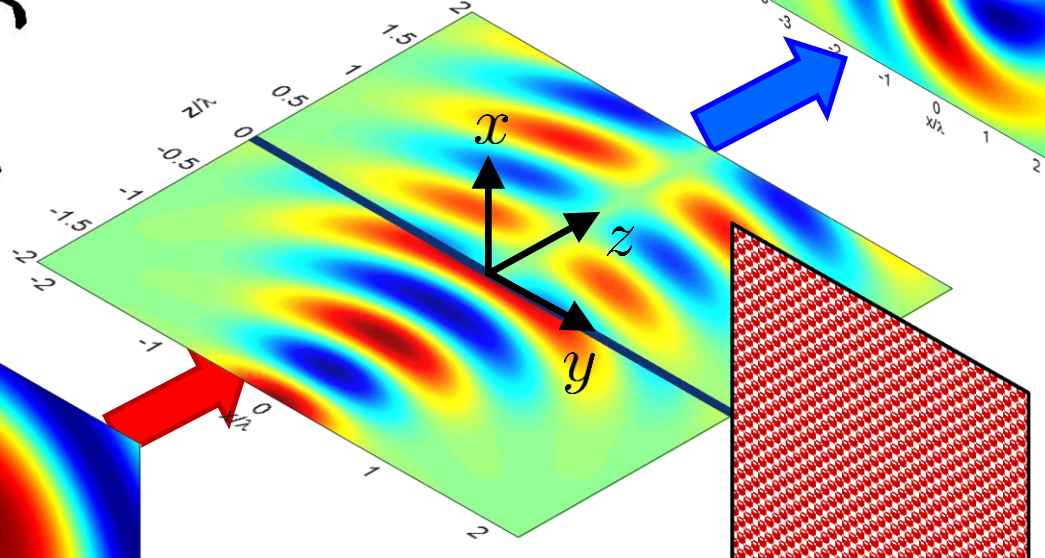
Metasurface Synthesis Problem

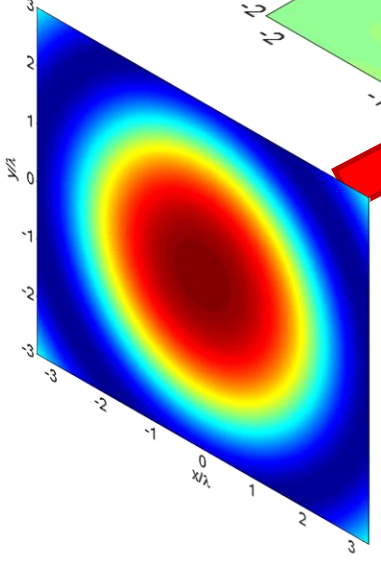


Metasurface ?



Output Wave ✓





Input Wave ✓

SYNTHESIS → TWO-STEPS:

- **Metasurface "transfer function"**
- **Scattering "molecules"**

Goals:

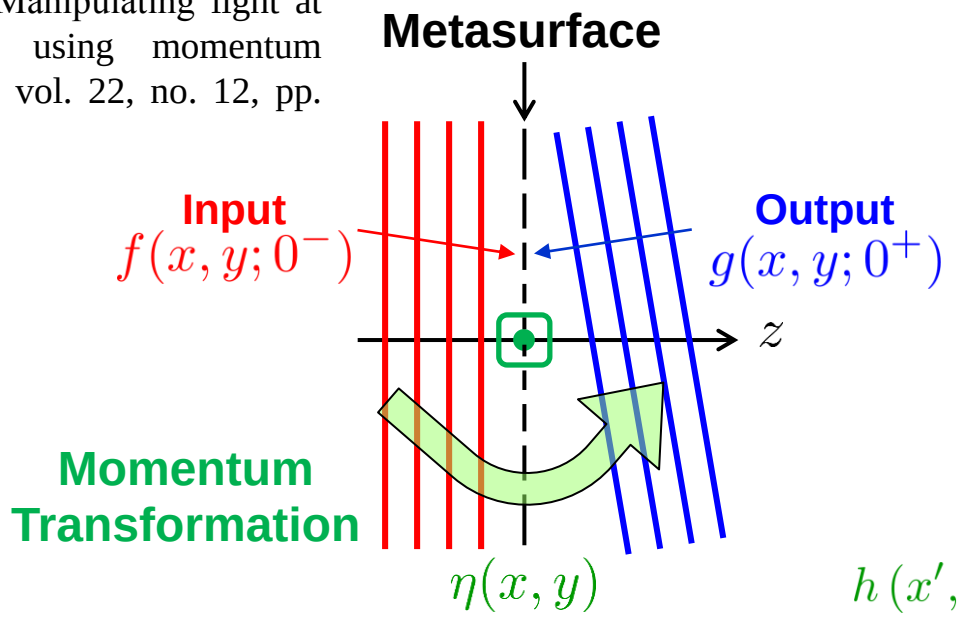
- Systematic/universal
- Scalar, paraxial, vector
- Infinite & truncated MS
- PW & any other wave
- Manipulation at distance
- Easy to Implement

Benefits:

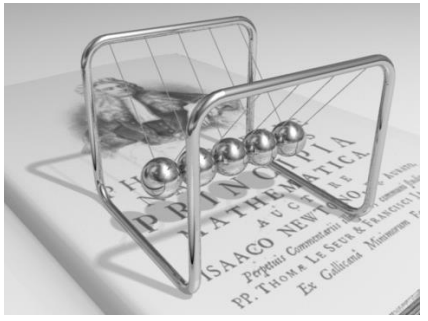
- Physical insight
- Inspiring new devices

Momentum Transformation Method (Spectral)

M. A. Salem and C. Caloz, "Manipulating light at distance by a metasurface using momentum transformation," ***Opt. Express***, vol. 22, no. 12, pp. 14530–14543, Jun. 2014.



Momentum Conservation



$$h(x',y';x,y) \neq h(x-x',y-y')$$

Space shift-variance: $g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')\, h(x',y';x,y)\, dx'dy'$ **Green function**

Zero EM thickness ⇒ locality: $h(x',y';x,y) = \delta(x'-x,y'-y)\,\eta(x',y')$

Consequence of locality: $g(x,y) = f(x,y)\,\eta(x,y)$

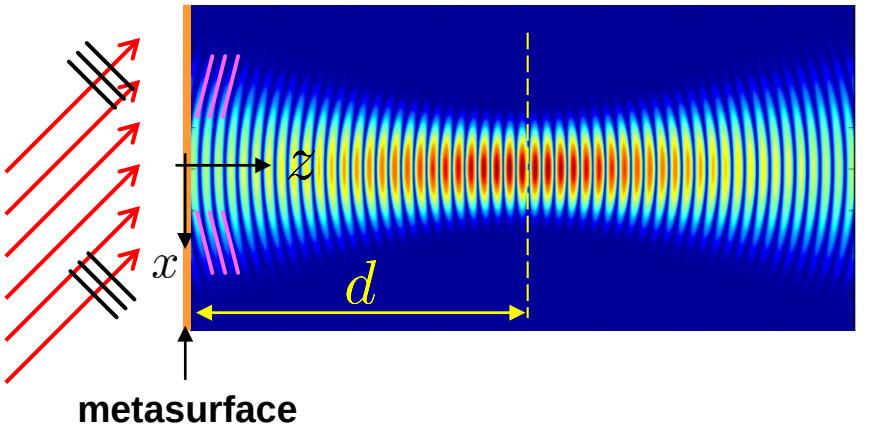
Fourier transform: $\tilde{g}(k_x,k_y) = \tilde{f}(k_x,k_y) * \tilde{\eta}(k_x,k_y)$

Momentum shift-invariance ≡ momentum conservation: $\tilde{f}(k_x-K_x,k_y-K_y) \overset{z=0}{\rightarrow} \tilde{g}(k_x-K_x,k_y-K_y)$

MT Scalar Case Closed-Form Example

$$\tilde{\eta}\left(k_x,k_y\right)=\tilde{g}\left(k_x,k_y\right)\tilde{\varphi}\left(k_x,k_y;k,d\right)*\tilde{\xi}\left(k_x,k_y\right)$$

Goal: π/4 oblique PW → Gaussian beam w/o reflection



$$f\left(x,y,z\right)=e^{ik(x+z)/\sqrt{2}}$$

$$\xi(x,y,0)=1/f(x,y,0)=e^{-ikx/\sqrt{2}}$$

$$g\left(x,y,z\right)=\frac{w_0}{w\left(z-d\right)}\exp\left[-\frac{x^2+y^2}{w^2\left(z-d\right)}-ik\left(z-d+\frac{x^2+y^2}{2R\left(z-d\right)}\right)\right]$$

$$w(z)=w_0\sqrt{1+(z/z_R)^2}$$
$$R(z)=z\left[1+(z_R/z)^2\right]$$
$$z_R=\pi w_0^2/\lambda$$
$$w_0=2\lambda$$

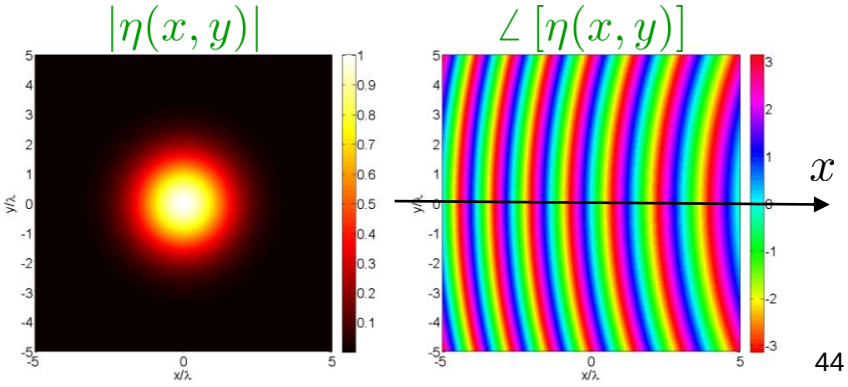
$$\tilde{\xi}\left(k_x,k_y\right)=\mathcal{F}_{xy}\{\xi(x,y,0)\}=\delta(k_x+k/\sqrt{2})\delta(k_y)$$

$$\tilde{g}\left(k_x,k_y\right)=\mathcal{F}_{xy}\{g(x,y,0)\}=\pi^2 w_0\exp\left[-\pi^2 w(-d)^2[k_x^2+k_y^2]+ikd+i2[k_x^2+k_y^2]\pi^2 R(-d)/k\right]$$

$$\tilde{\varphi}\left(k_x,k_y;k,d\right)=e^{-i\sqrt{k^2-k_x^2-k_y^2}d}$$

$$\tilde{\eta}\left(k_x,k_y\right)=$$
$$\pi^2 w_0 e^{\pi^2 w_0^2[1+d^2/z_R^2]k^2/2+ikd-ik\pi^2 d[1+z_R^2/d^2]/2}e^{-i\sqrt{3}kd/2}$$

$$\eta\left(x,y\right)=\mathcal{F}_{xy}^{-1}\left\{\tilde{\eta}(k_x,k_y)\right\}$$

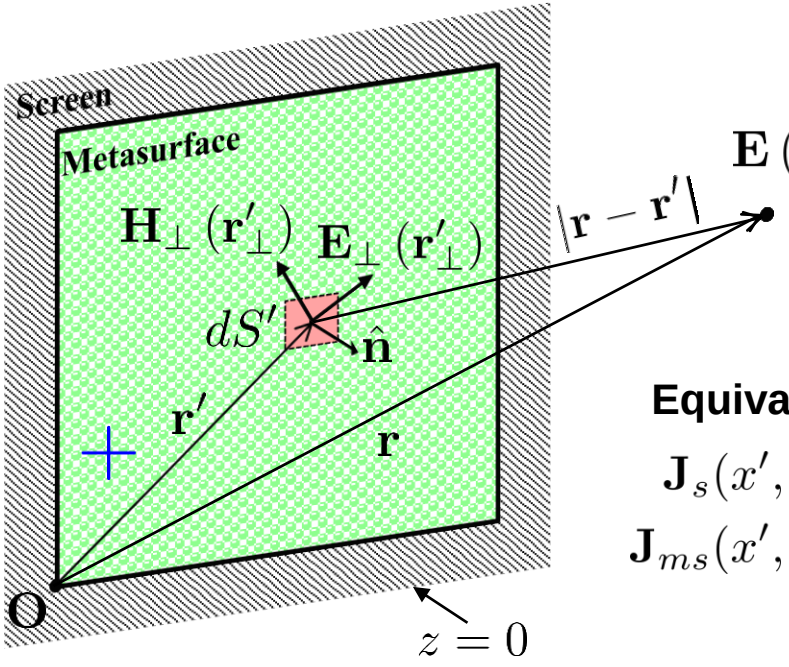


Vector Case 1/2

$$\mathbf{E}_{\perp}^{\overline{+}}(\mathbf{r}'_{\perp}) = \mathbf{E}(x, y, 0_{\overline{+}}) \times \hat{\mathbf{n}}$$
$$\mathbf{H}_{\perp}^{\overline{+}}(\mathbf{r}'_{\perp}) = \mathbf{H}(x, y, 0_{\overline{+}}) \times \hat{\mathbf{n}}$$

$z = 0_- \rightarrow f(x, y)$

$z = 0_+ \rightarrow g(x, y)$



Equivalent Currents:

$$\mathbf{J}_s(x', y') = \hat{\mathbf{n}} \times \mathbf{H}_{\perp}(x', y'; z = 0)$$
$$\mathbf{J}_{ms}(x', y') = -\hat{\mathbf{n}} \times \mathbf{E}_{\perp}(x', y'; z = 0)$$

Radiated fields in terms of el./mag. equivalent currents:

$$\mathbf{E}(\mathbf{r}) = \frac{i}{\omega\epsilon} \int_A k^2 \mathbf{J}_s G + [\mathbf{J}_s \cdot \nabla'] \nabla' G + i\omega\epsilon \mathbf{J}_{ms} \times \nabla' G dS'$$
$$\mathbf{H}(\mathbf{r}) = \frac{i}{\omega\mu} \int_A k^2 \mathbf{J}_{ms} G + [\mathbf{J}_{ms} \cdot \nabla'] \nabla' G - i\omega\mu \mathbf{J}_s \times \nabla' G dS'$$

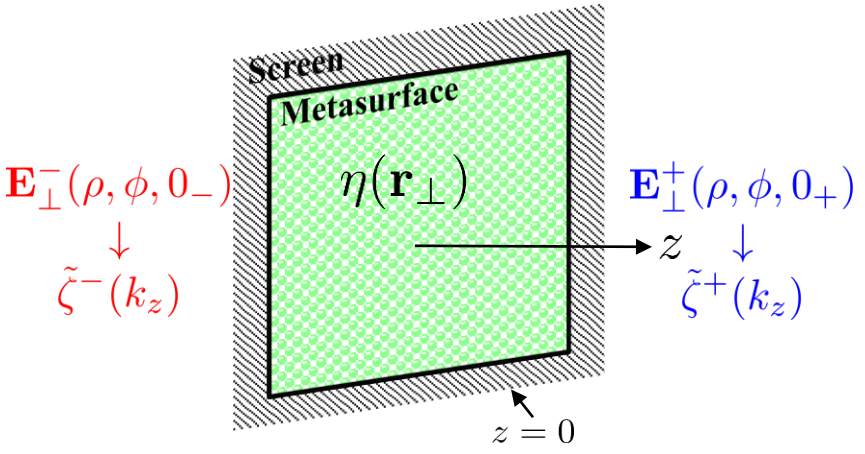
Free-space scalar Green function: $G = G(\mathbf{r}, \mathbf{r}') = \dfrac{e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}}{4\pi |\mathbf{r} - \mathbf{r}'|}$

Radiated fields in terms of <u>known</u> fields:

$$\mathbf{E} = \frac{i}{\omega\epsilon} \int_A k^2 [\hat{\mathbf{n}} \times \mathbf{H}_{\perp}] G + [\hat{\mathbf{n}} \times \mathbf{H}_{\perp}] \cdot \nabla' \nabla' G - i\omega\epsilon [\hat{\mathbf{n}} \times \mathbf{E}_{\perp}] \times \nabla' G dS'$$
$$\mathbf{H} = \frac{i}{\omega\mu} \int_A -k^2 [\hat{\mathbf{n}} \times \mathbf{E}_{\perp}] G - [\hat{\mathbf{n}} \times \mathbf{E}_{\perp}] \cdot \nabla' \nabla' G - i\omega\mu [\hat{\mathbf{n}} \times \mathbf{H}_{\perp}] \times \nabla' G dS'$$

Vector Case 2/2

VBB basis function → $\sum_{n=-\infty}^{\infty} \int_0^{\infty} \mathbf{E}_{\perp}^{\mathrm{VBB}}(\rho,\phi;k_z) \times \mathbf{H}_{\perp}^{*\mathrm{VBB}}(\rho,\phi;k_z')\rho d\rho = P_{k_z}^{\mathrm{VBB}}\delta(k_z - k_z')$



k_z eigenmode expansion:

$$\mathbf{E}^{\mp}(\rho,\phi;z) = \int_0^{+\infty} \tilde{\zeta}^{\mp}(k_z)\mathbf{E}_{\perp}^{\mathrm{VBB}}(\rho,\phi;k_z)\, e^{ik_z z} dk_z$$

$$\sum_{n=-\infty}^{\infty} \int_0^{\infty} \ldots \times \mathbf{H}^{*\mathrm{VBB}\mp}(\rho,\phi;k_z)\rho d\rho$$

& VBB orthogonality

Scalar expansion coefficients:

$$\tilde{\zeta}^-(k_z) = \frac{1}{P_{k_z}^{\mathrm{VPW}}} \sum_{n=-\infty}^{\infty} \int_0^{\infty} \mathbf{E}_{\perp}^{-}(\rho,\phi) \times \mathbf{H}_{\perp}^{*\mathrm{VBB}}(\rho,\phi;k_z)\rho d\rho$$

$$\tilde{\zeta}^+(k_z) = \frac{1}{P_{k_z}^{\mathrm{VPW}}} \sum_{n=-\infty}^{\infty} \int_0^{\infty} \mathbf{E}_{\perp}^{+}(\rho,\phi) \times \mathbf{H}_{\perp}^{*\mathrm{VBB}}(\rho,\phi;k_z)\rho d\rho$$

Reverse propagator: $\tilde{\varphi}(k_z,d) = e^{-ik_z d}$

Scalarized (correspondence with scalar case) solution: $\tilde{\zeta}^-(k_z) * \tilde{\eta}(k_z) = \tilde{\zeta}^+(k_z)\tilde{\varphi}(k_z,d)$

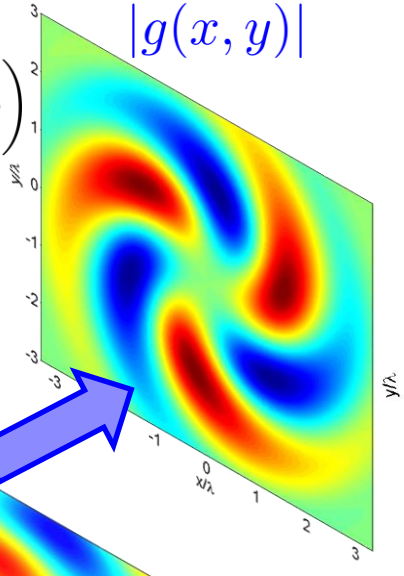
Modal transformation, using $k^2 = k_\rho^2 + k_z^2$ **:** $\tilde{\eta}(k_z) \Rightarrow \eta(x,y)$

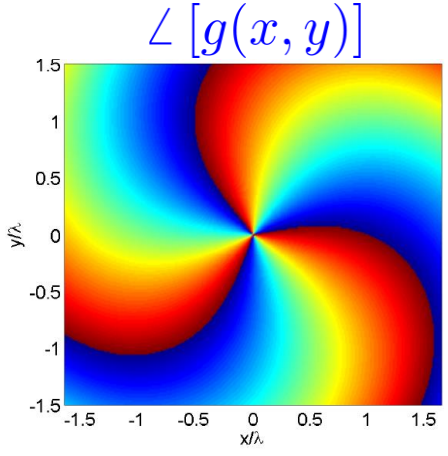
Example 1: Stable Vortex Beam Generation

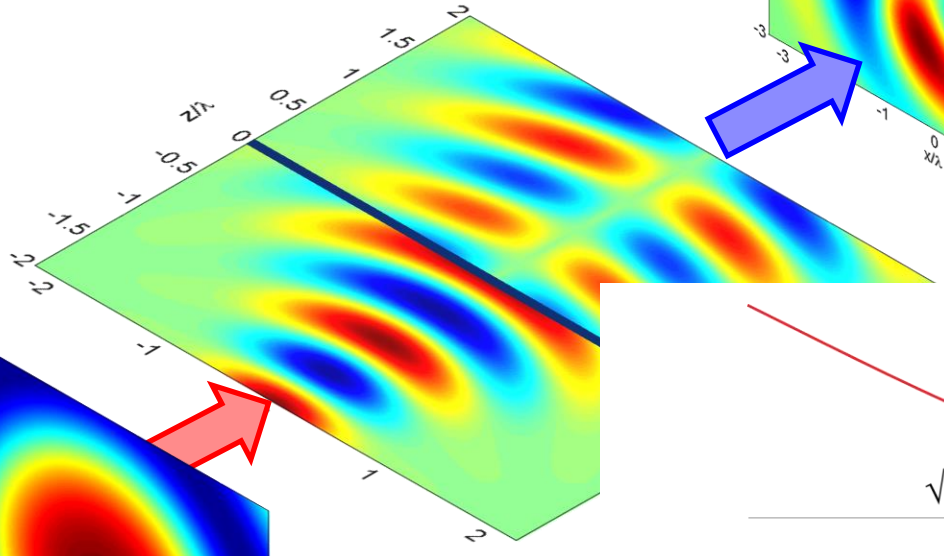
Output Wave

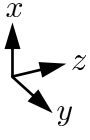
(vortex hypergeometric Gaussian)

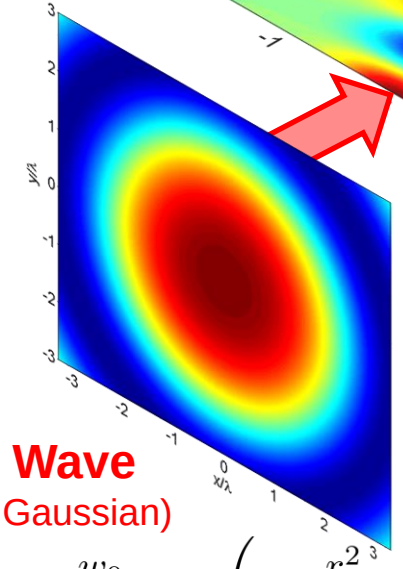
$$g(x,y,z) = \frac{i^{|m|+1}\xi^{p/2}}{[\xi+i]^{1+|m|+p/2}}\left[\frac{\rho}{w_0}\right]^{|m|}\exp\left(\frac{-i\rho^2}{w_0^2[\xi+i]}+im\phi\right)$$
$$\times\frac{\Gamma\left(1+|m|+\frac{p}{2}\right)}{\Gamma(|m|+1)}\,{}_1F_1\left(-\frac{p}{2},|m|+1;\frac{\rho^2}{w_0^2\xi[\xi-i]}\right)$$









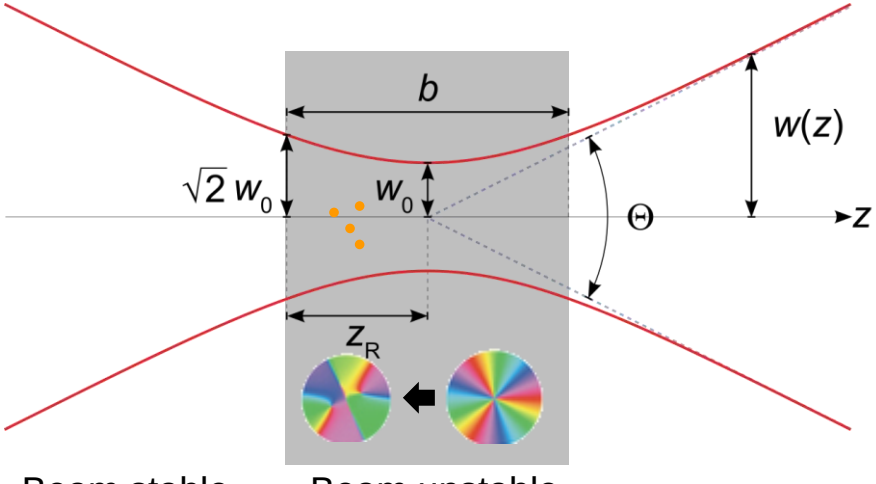


$|f(x,y)|$

Input Wave

(ordinary Gaussian)

$$f(x,y,z) = \frac{w_0}{w(z)}\exp\left(-\frac{r^2}{w(z)^2}-ik\left[z+\frac{r^2}{2R(z)}\right]-i\vartheta(z)\right)$$



Beam stable

Beam unstable

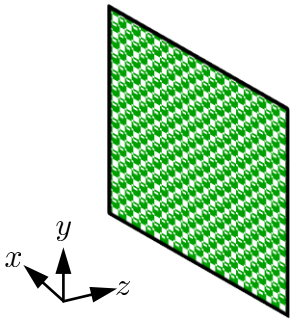
Small perturbation (e.g dust particles)

Degeneration of OAM modes

Example 2: Delay-Start Airy Beam Generation

Input Wave
(normally incident plane wave)

$$f(z) = e^{ikz}$$



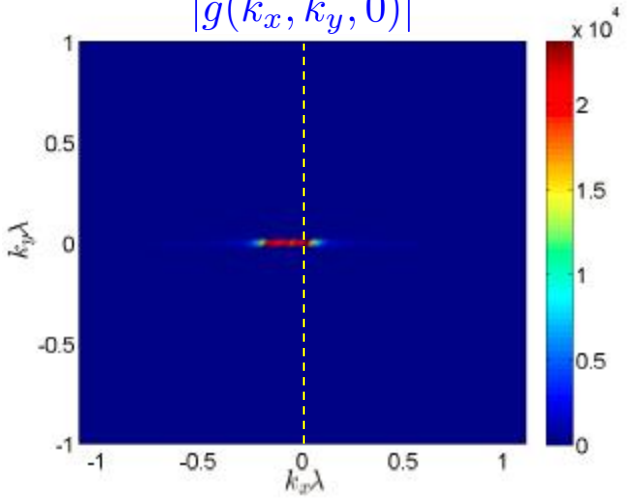
Output Wave (Airy beam with start at $z = d$)

$$g(x,z) = \mathrm{Ai}\left(s - \frac{\zeta^2}{4}\right)\exp\left(i\left[\frac{s\zeta}{2} - \frac{\zeta^3}{12}\right]\right)$$

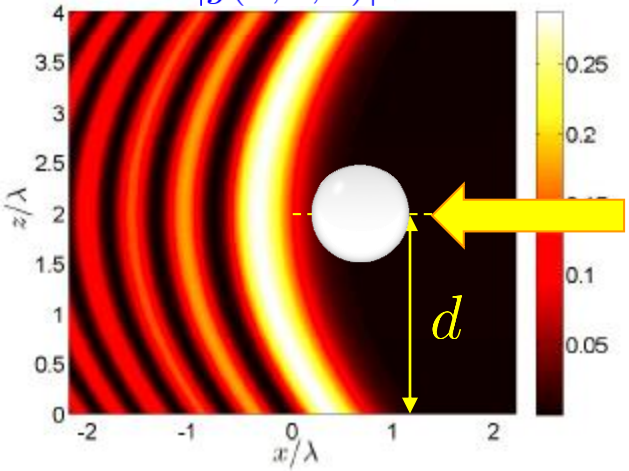
$$\tilde{\varphi}(k_x;k,d) = \exp\left(-i\sqrt{k^2 - k_x^2}\,d\right)$$

$$s = x/x_0,\ \zeta = z/kx_0^2$$

$|g(k_x,k_y,0)|$

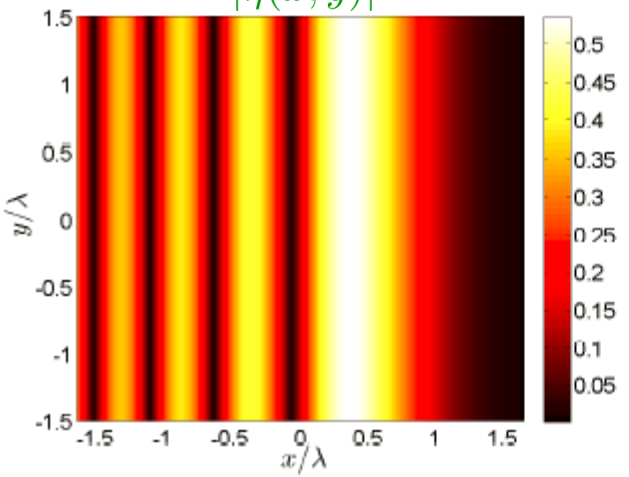


$|g(x,0,z)|^2$

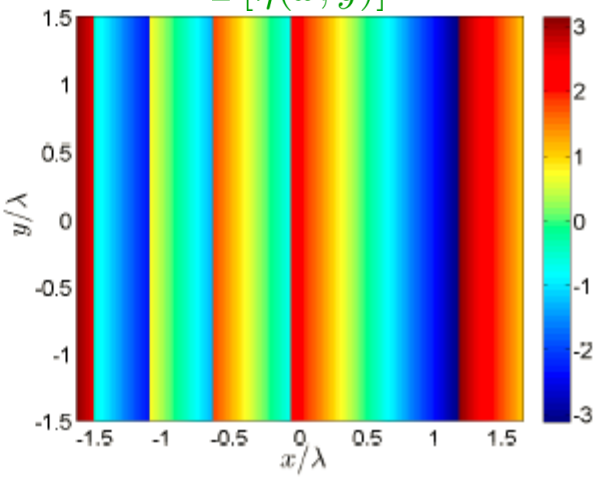


Curve around object at $z = d$

$|\eta(x,y)|$



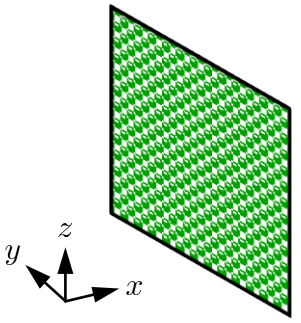
$\angle[\eta(x,y)]$



Example 3: Pencil Beam Generation

Input Wave
(normally incident plane wave)

$$\mathbf{E}(z) = \hat{\mathbf{x}} e^{ikz}$$



Output Wave (far-field pencil beam)

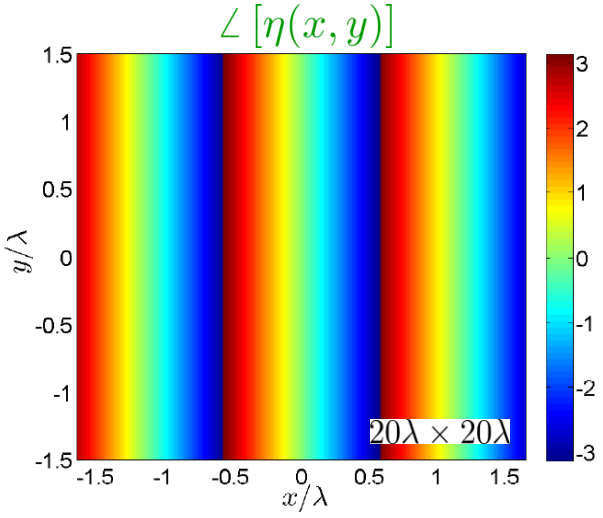
$$\mathbf{E}(\theta,\phi) = -j\omega\mu\mathbf{A}(\theta,\phi)$$

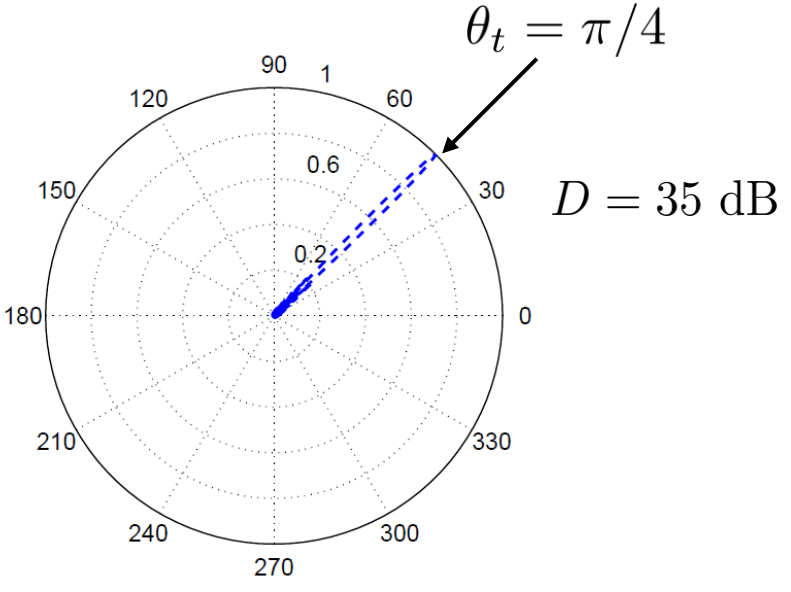
$$= -j\omega\mu \iint_S \mathbf{J}(x',y') \frac{e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}}{4\pi} d\mathbf{r}'_\perp$$

$$(\theta_t, \phi_t) = (\pi/4, 0), \quad \Delta\Omega \approx \pi/64$$

$$\Delta\Omega = 2\pi \int_0^\pi \frac{|AF(\theta)|^2}{|AF(\theta_t)|^2} \sin(\theta)\, d\theta,$$

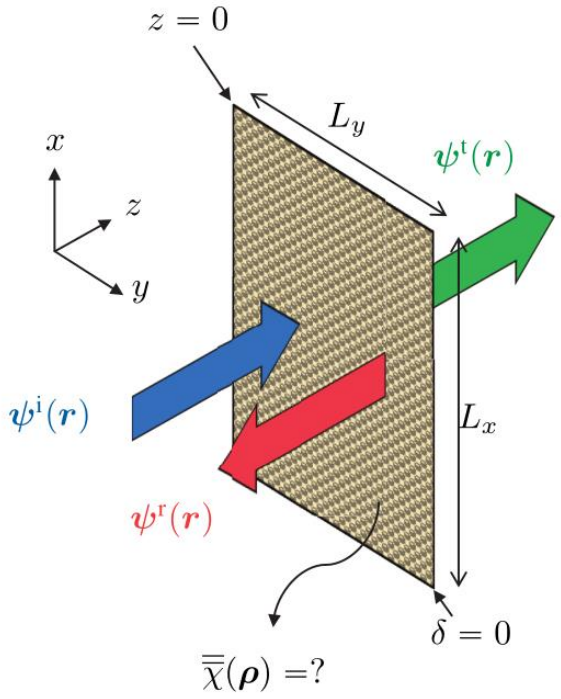
Supergain device with unique source





$D = 35$ dB

Susceptibility Method (Spatial)



Generalized Sheet Transition Conditions (GSTCs)*

$$\hat{z} \times \Delta \boldsymbol{H} = j\omega \boldsymbol{P}_{\parallel} - \hat{z} \times \nabla_{\parallel} M_z$$

$$\Delta \boldsymbol{E} \times \hat{z} = j\omega\mu \boldsymbol{M}_{\parallel} - \nabla_{\parallel} \left(\frac{P_z}{\epsilon} \right) \times \hat{z}$$

Difference Fields:

$$\Delta \boldsymbol{\Psi} = \Delta \boldsymbol{\Psi}(\boldsymbol{\rho})\Big|_{z=0^-}^{0^+} = \boldsymbol{\Psi}^{\mathrm{t}} - \left(\boldsymbol{\Psi}^{\mathrm{i}} + \boldsymbol{\Psi}^{\mathrm{r}} \right), \quad \boldsymbol{\Psi} = \boldsymbol{H}, \boldsymbol{E}$$

Bianisotropic Metasurface (Surf. Pol.)

$$\boldsymbol{P} = \epsilon N \langle \overline{\overline{\alpha}}_{\mathrm{ee}} \rangle \boldsymbol{E}_{\mathrm{act}} + \sqrt{\mu\epsilon} N \langle \overline{\overline{\alpha}}_{\mathrm{em}} \rangle \boldsymbol{H}_{\mathrm{act}},$$

$$\boldsymbol{M} = N \langle \overline{\overline{\alpha}}_{\mathrm{mm}} \rangle \boldsymbol{H}_{\mathrm{act}} + \sqrt{\frac{\epsilon}{\mu}} N \langle \overline{\overline{\alpha}}_{\mathrm{mm}} \rangle \boldsymbol{E}_{\mathrm{act}},$$

Disk Particle Modeling (Surf. Susc.)

$$\boldsymbol{P} = \epsilon \overline{\overline{\chi}}_{\mathrm{ee}} \boldsymbol{E}_{\mathrm{av}} + \overline{\overline{\chi}}_{\mathrm{em}} \sqrt{\mu\epsilon} \boldsymbol{H}_{\mathrm{av}},$$

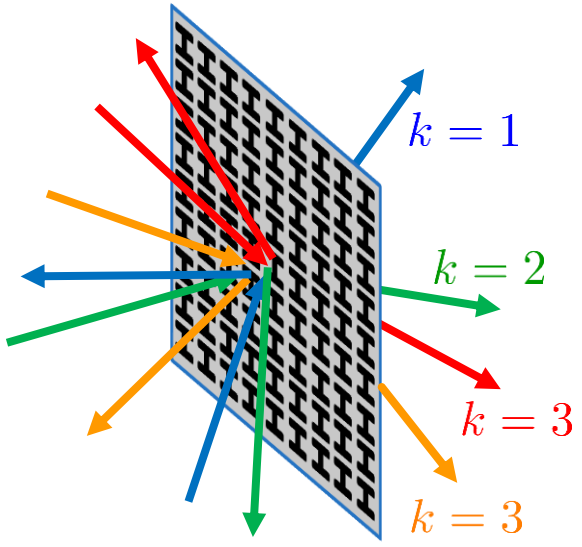
$$\boldsymbol{M} = \overline{\overline{\chi}}_{\mathrm{mm}} \boldsymbol{H}_{\mathrm{av}} + \overline{\overline{\chi}}_{\mathrm{me}} \sqrt{\frac{\epsilon}{\mu}} \boldsymbol{E}_{\mathrm{av}},$$

Average Fields: $\boldsymbol{\Psi}_{\mathrm{av}} = \boldsymbol{\Psi}_{\mathrm{av}}(\boldsymbol{\rho}) = \dfrac{\boldsymbol{\Psi}^{\mathrm{t}} + \left(\boldsymbol{\Psi}^{\mathrm{i}} + \boldsymbol{\Psi}^{\mathrm{r}} \right)}{2}$

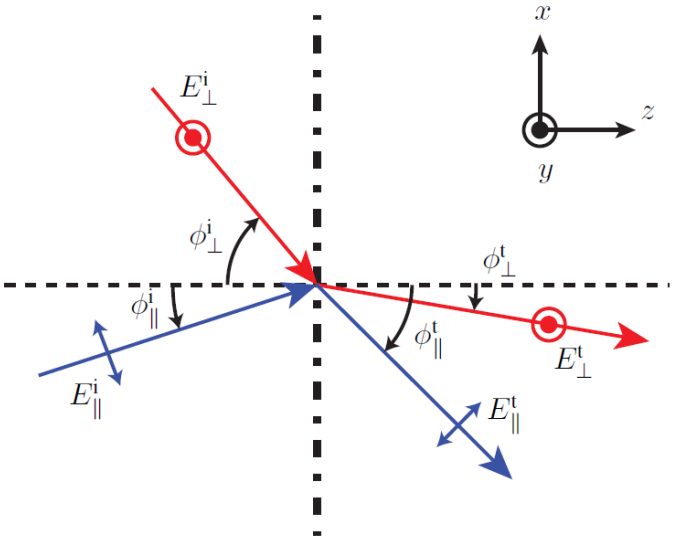
K. Achouri, M. A. Salem, and C. Caloz, "General metasurface synthesis based on susceptibility tensors," *arXiv:1408.0273*.

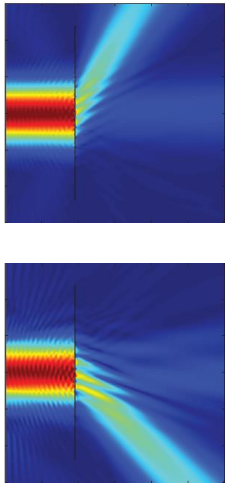
Examples: Playing with Degrees of Freedom !

16 χ's / 16 eqs ⇒ 4 Independent Wave Triplets

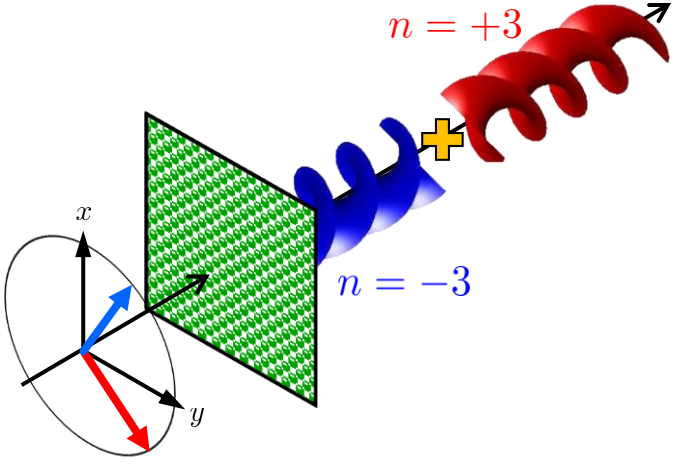


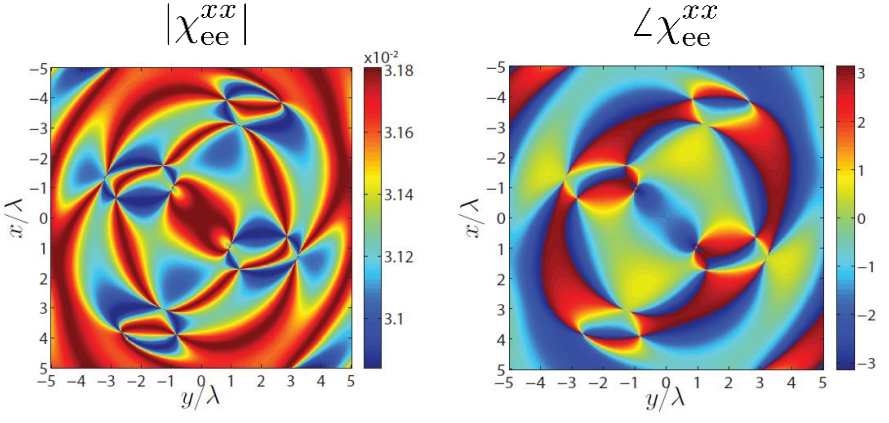
Refractive Birefringence (8 χ's)





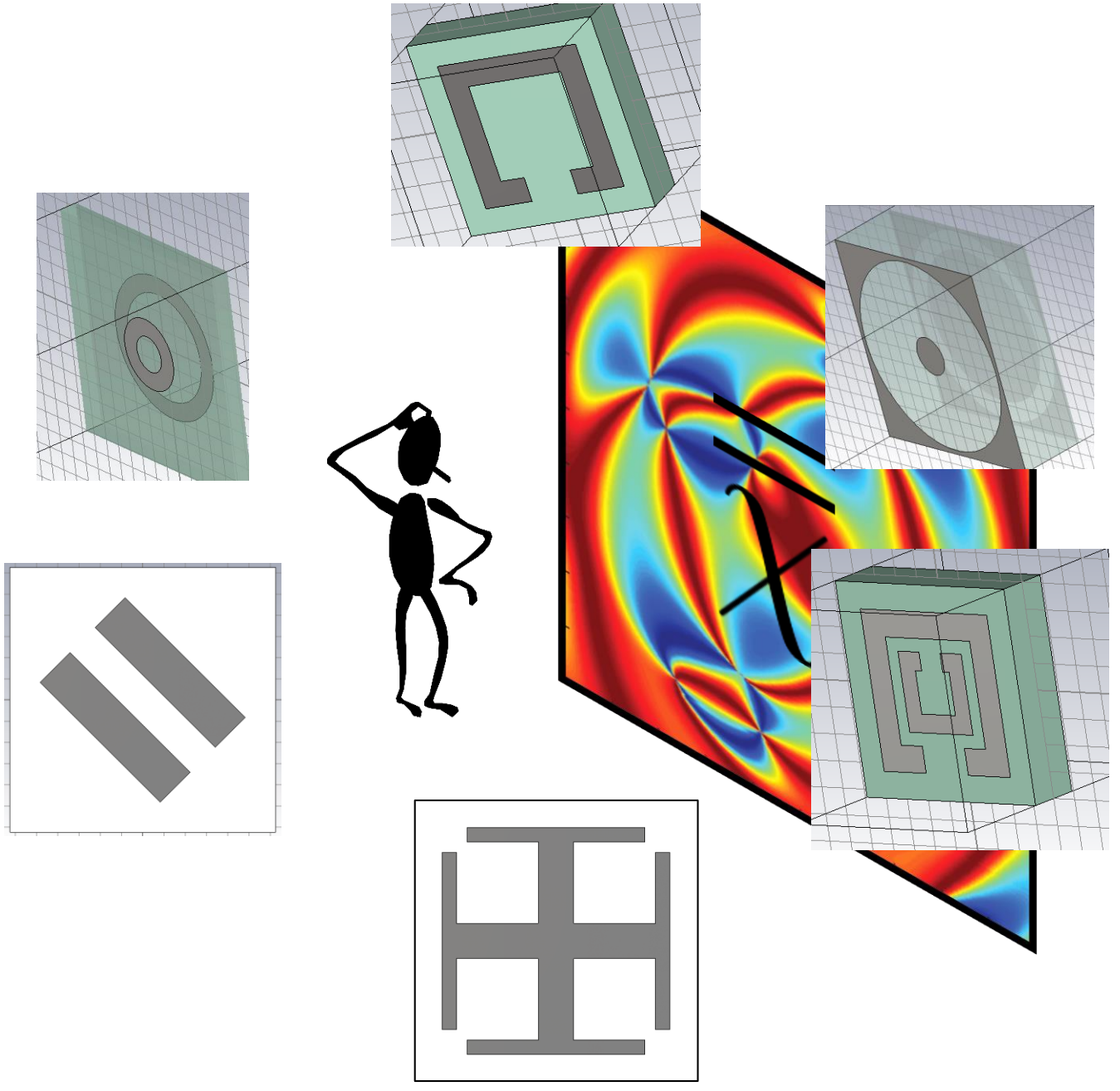
Orbital Angular Momentum (OAM) Multiplexing





K. Achouri, M. A. Salem, and C. Caloz, "General metasurface synthesis based on susceptibility tensors," *arXiv:1408.0273*.

How to Find the Scatterers for Synthesized $\chi(x,y)$?

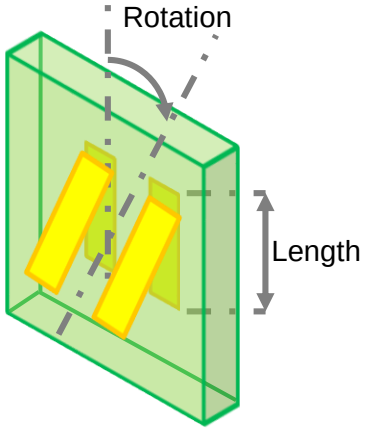


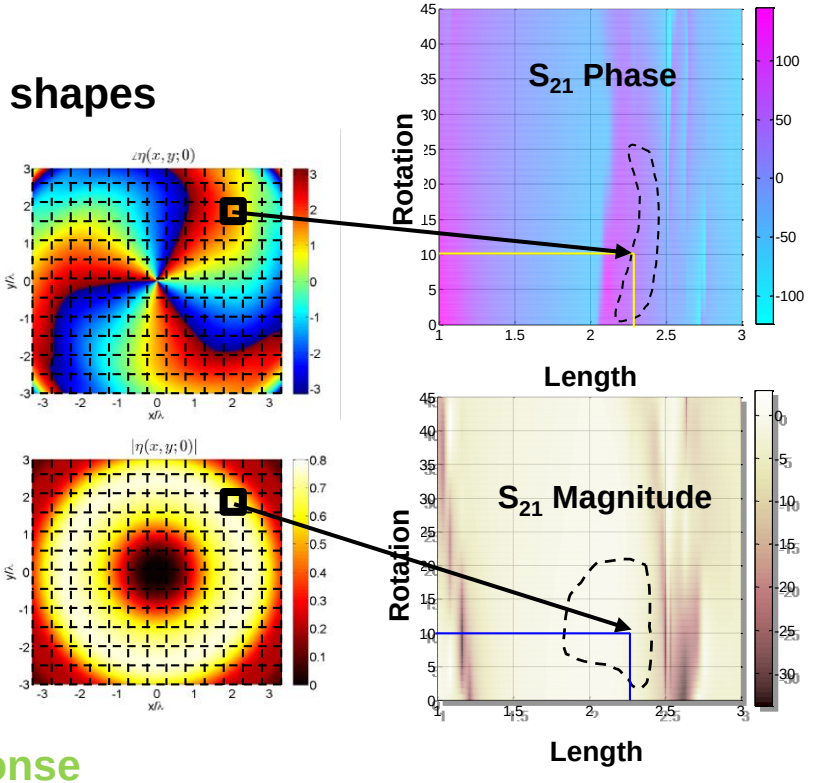
Design & Limitations

❖ **No systematic way to determine scattering element shapes**

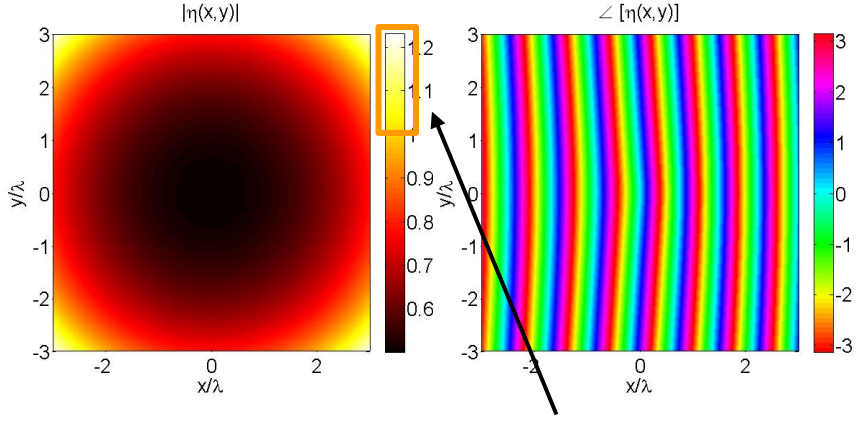
Element shape via **parametric mapping**

Scattering element



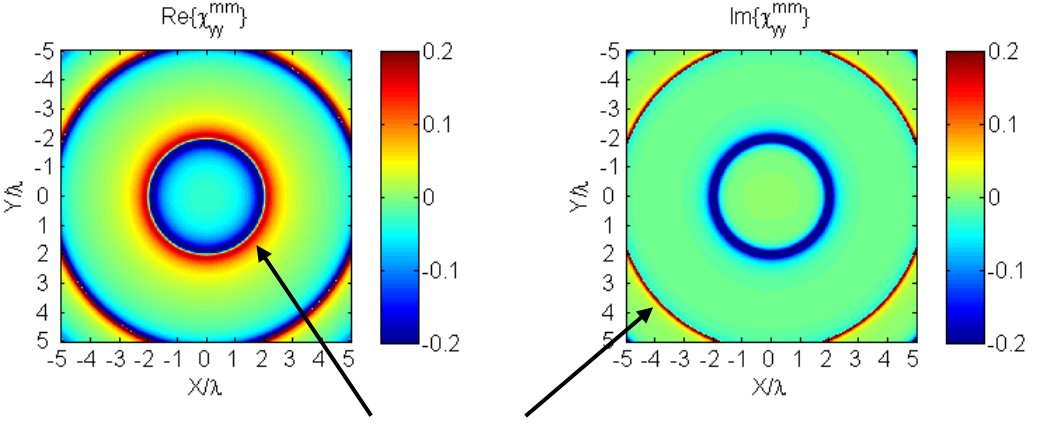


❖ **Elements may be active or have non-physical response**



Transmission **larger than unity**

(normal Gaussian to oblique PW)



Susceptibility **varies too fast**

(Dipole field to normal PW)

The Space DE – Metasurfaces QUIZ

1) What us the difference between metasurfaces and FSS ?

   As MTMs vs ADs: same config., but more properties + material approach

2) When was the first gyrotropic nonreciprocal metasurface reported ?

   July 2011, (T. Kodera, D. L. Sounas, and C. Caloz,
   Appl. Phys. Lett., vol. 99, p. 031114)

3) Does the Science paper « Generalized Refraction » do the job ?

   Mmhh, NOT PERFECTLY, parasitic refraction & reflection

4) What is the different between Moment Method and OTF?

   SHIFT INVARIANCE IN K instead of r

5) What are the 2 steps in metasurface synthesis ?

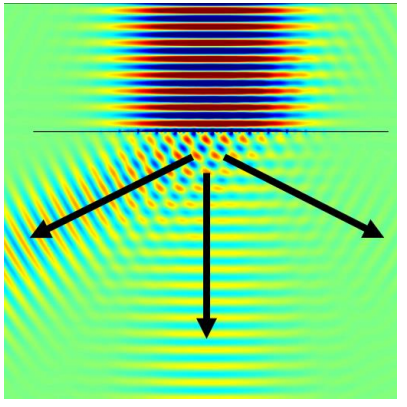
   1. Mathematical transfer function, 2. Scatterers

6) What is the greatest finding with the susceptibility method ?

   Possibibility of MULTIPLE TRANSFORMATIONS

7) What might be the hardest challenge in scatterer design ?

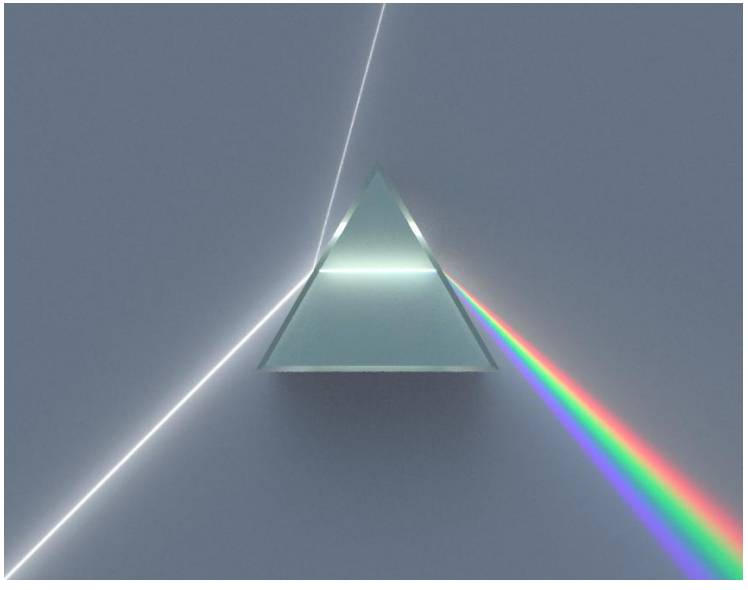
   Diffraction orders due to susceptibility approximation.



Outline

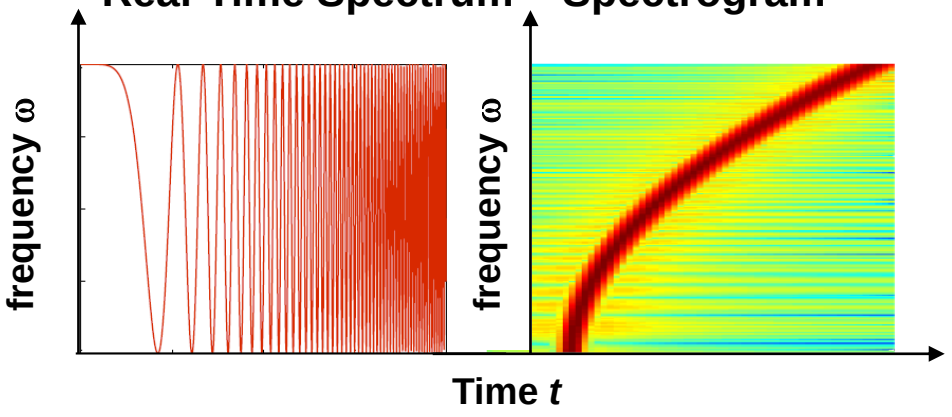
- I. The Metamaterial Enlightenment
- II. Time DE – Phasers
- III. Space DE – Metasurfaces
- IV. Space-Time DE – ST Processors
- V. What is our Future ?

ST Processing by Nature and Humans

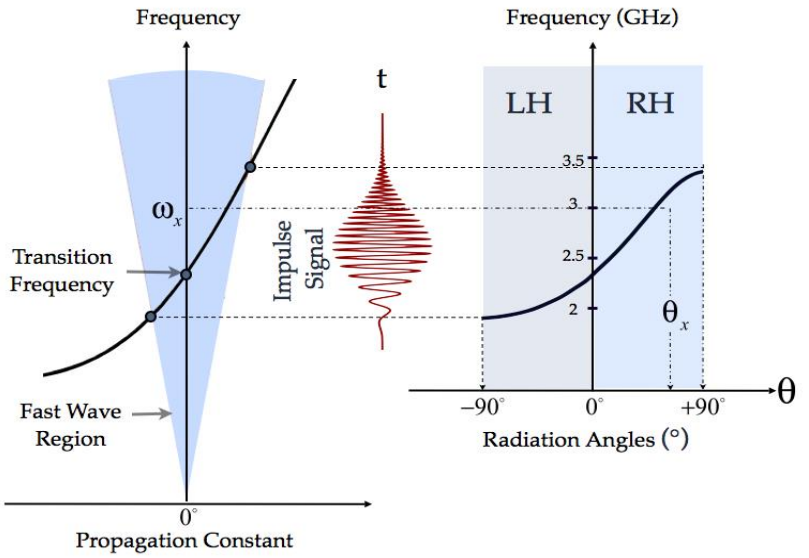


Real-Time Spectrum Analyzer (RTSA)

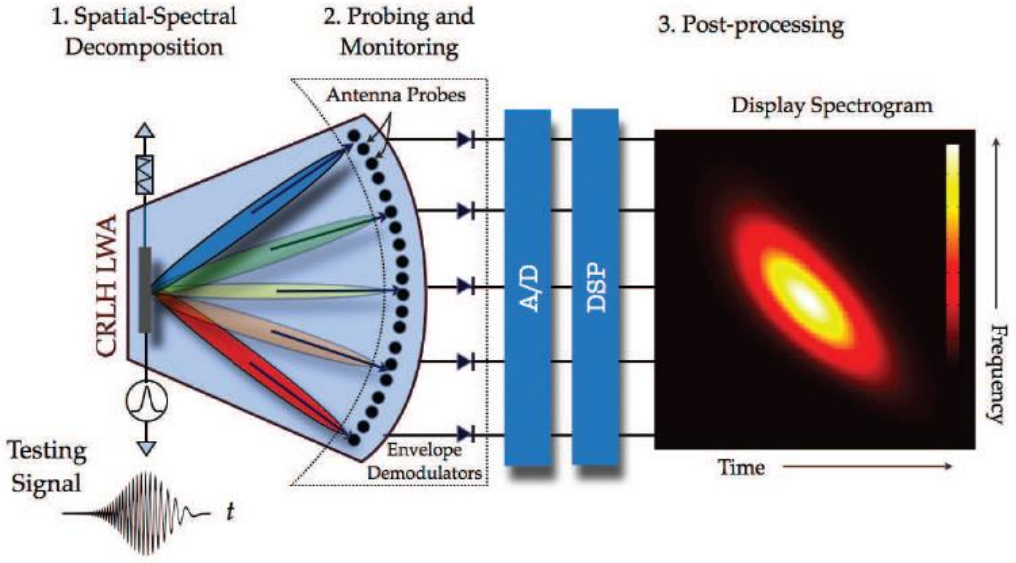
Real-Time Spectrum = Spectrogram



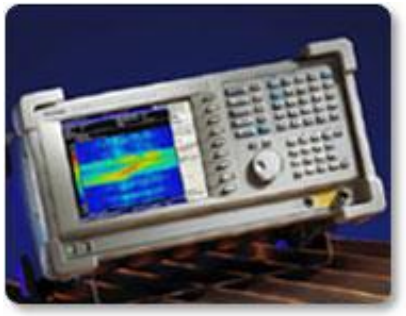
CRLH LWA Frequency-Space Mapping



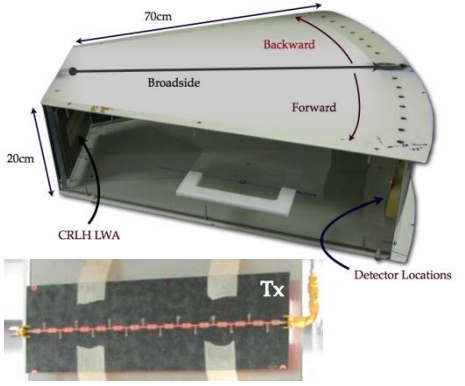
Principle



State of the Art (Tektronix RSA6000)



CRLH LWA Solution

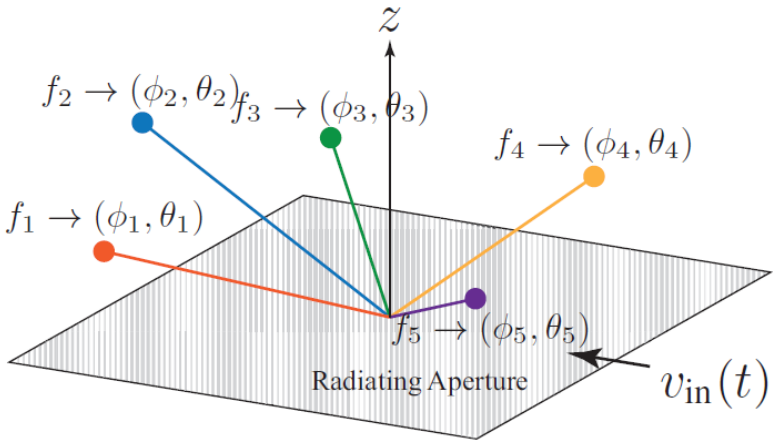


- Light post-processing
- Short acquisition time
- Broadband (UWB)
- ω-scalable to mm-wave

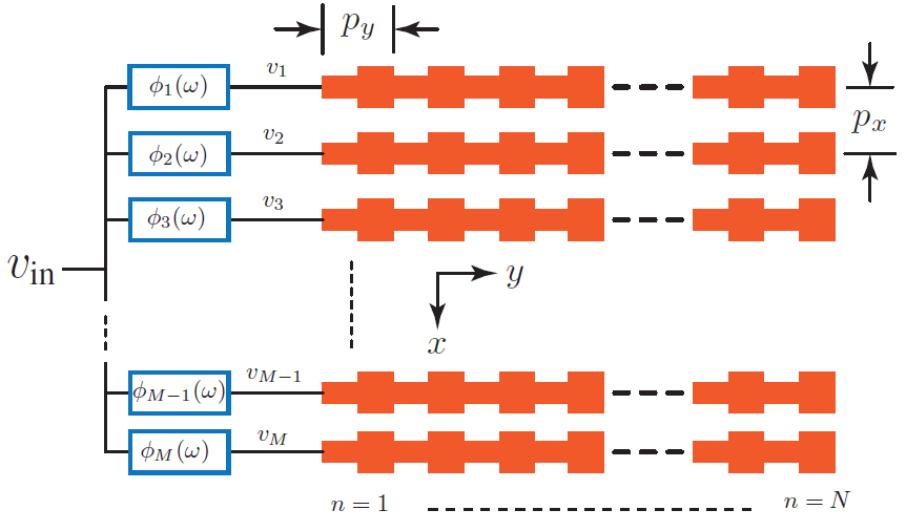
S. Gupta, S. Abielmona, and C. Caloz, "Microwave analog real-time spectrum analyzer (RTSA) based on the spatial-spectral decomposition property of leaky-wave structures," *IEEE Trans. Microw. Theory Tech.*, vol. 59, no. 12, pp. 2989–2999, Dec. 2009.

Ultra High-Speed Spectral Analysis using LW Array

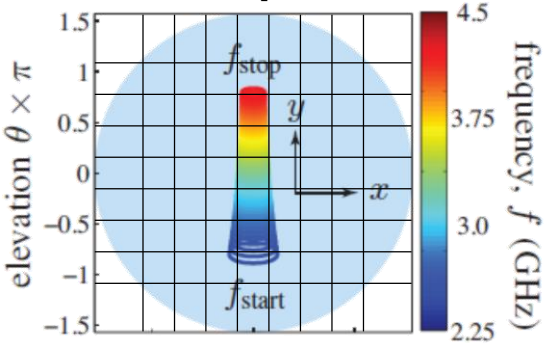
2D temporal-to-spatial frequency mapping



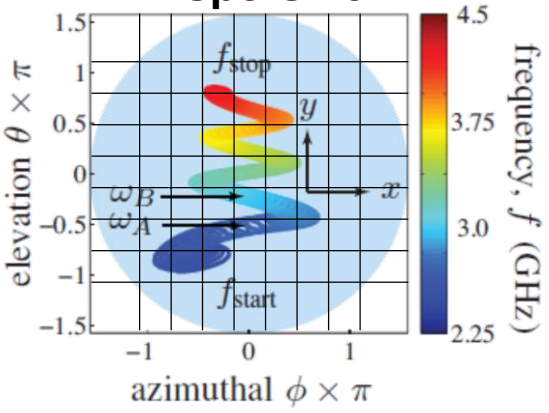
LWA Array with dispersive Feeding



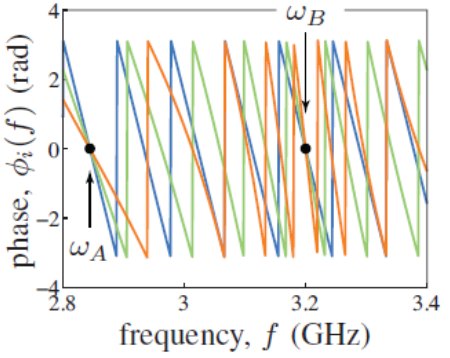
Non-dispersive



Dispersive



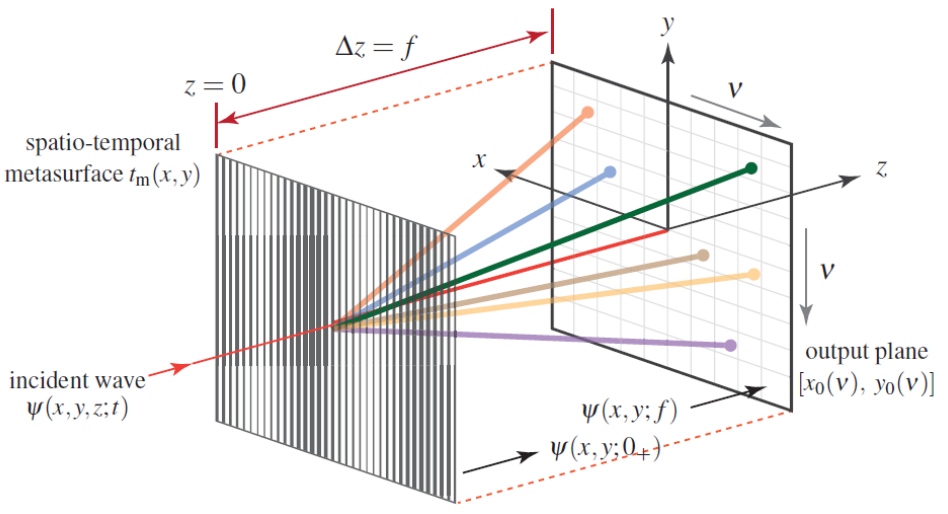
Dispersive Phase profile

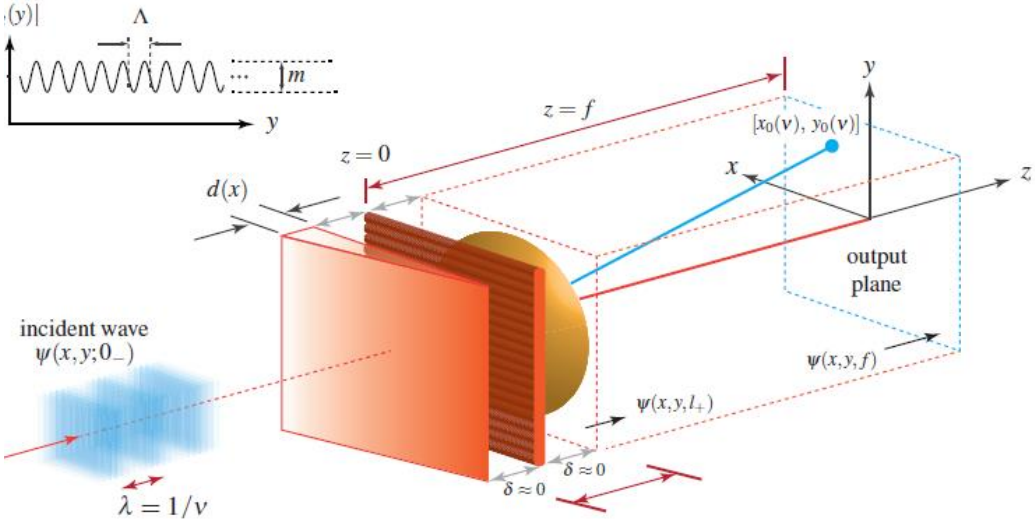


S. Gupta and C. Caloz, "Real–Time 2-D Spectral-Decomposition using a Leaky-Wave Antenna Array with Dispersive Feeding Network," IEEE APS/URSI 2015, *submitted*.

Spatio-Temporal Metasurface for 2D RT Spectral Analysis

Spatio-temporal Metasurface = **Dispersive Wedge** + **Diffraction Grating** + **Focusing Lens**



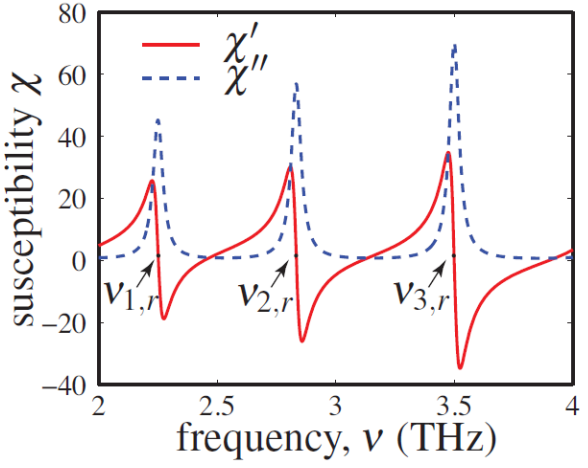


scanning Law

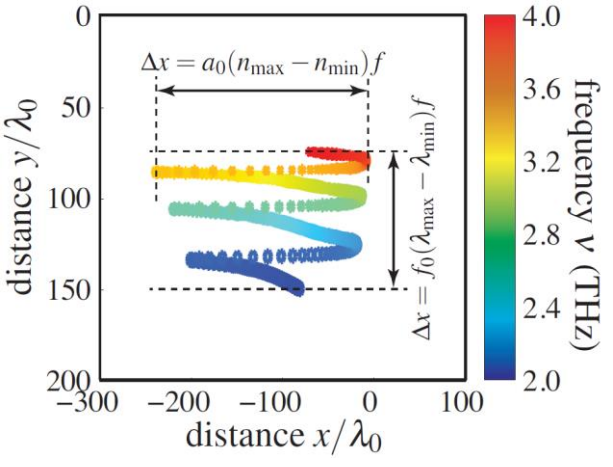
$$x_0(\nu) = a_0 n(\nu) f$$

$$y_0(\nu) = -f_0 \frac{c}{\nu} f$$

multi-resonant wedge



result



S. Gupta and C. Caloz, "Spatio-Temporal Metasurface for Real-time 2-D Spectrum Analysis ," arXiv:1412.7791, Dec. 2014.

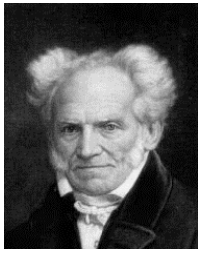
Space-Time (ST)

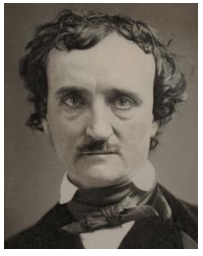
Religion

Philosophy

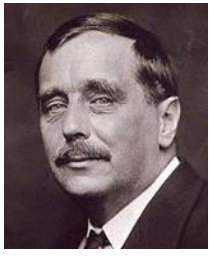
Literature

Incas


Schopenhauer

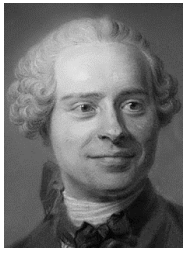

Poe

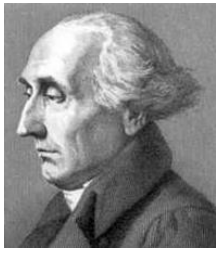
Proust


Wells

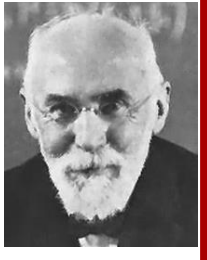
Mathematics

Physics


d'Alembert

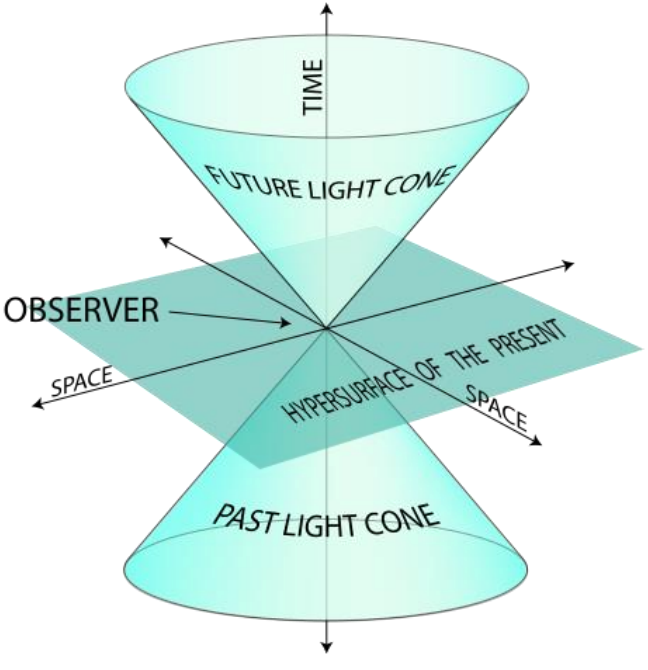

Lagrange

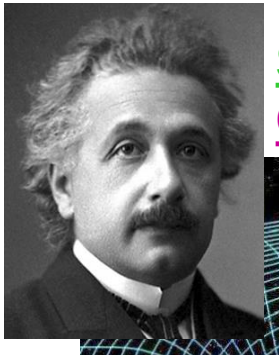
Cliffort


Lorentz

Minkowski

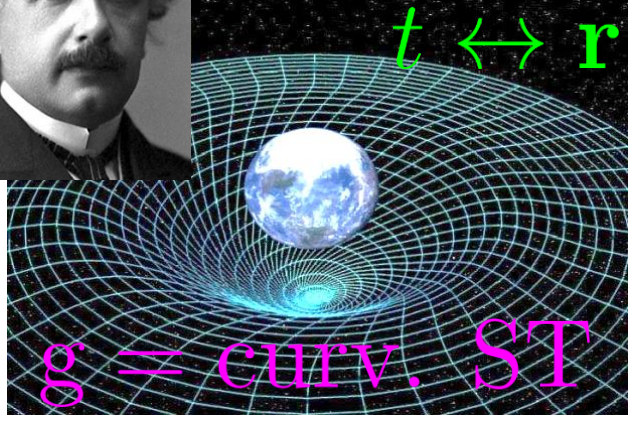
Minkoswki Space
(3D space +1D time)





Special Theory of Relativity

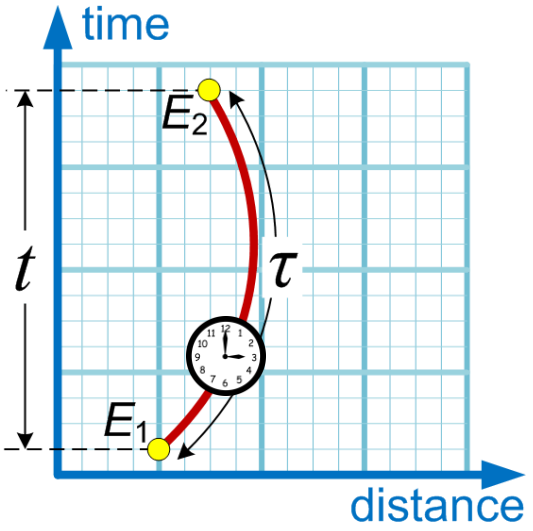
General Theory of Relativity



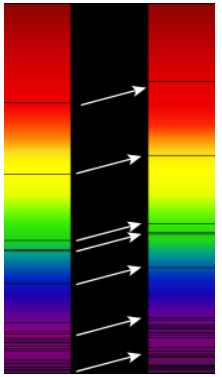
ST Interdependence in Electrodynamics

Proper (experienced) time:

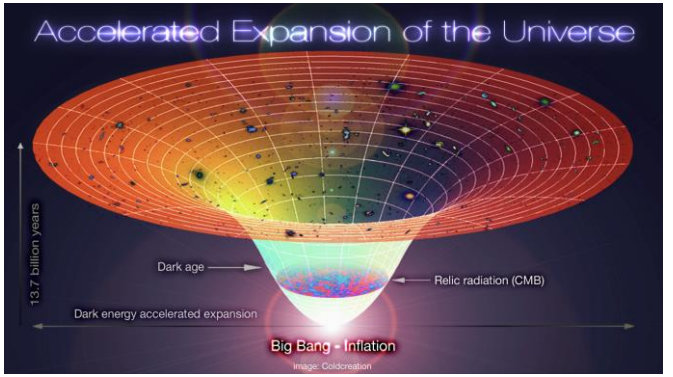
$$\tau = \int_0^t \sqrt{1 - \frac{v(t)}{c}}\, dt$$



Cosmological Red Shift



sun cluster



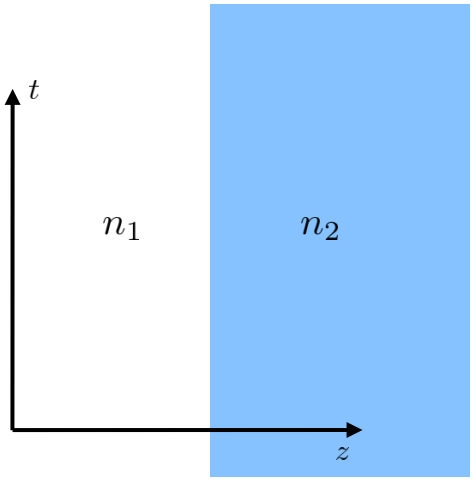
Related Effect: Doppler effect

$$\Omega = \left[1 - \frac{v_s(t)}{v_{ac}}\right] \omega$$

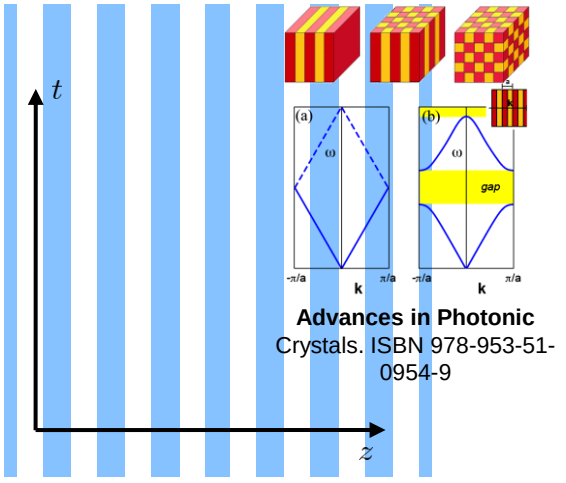
EM Space-Time Discontinuities

Images: public domain sources

Spatial and Spatiotemporal Structures

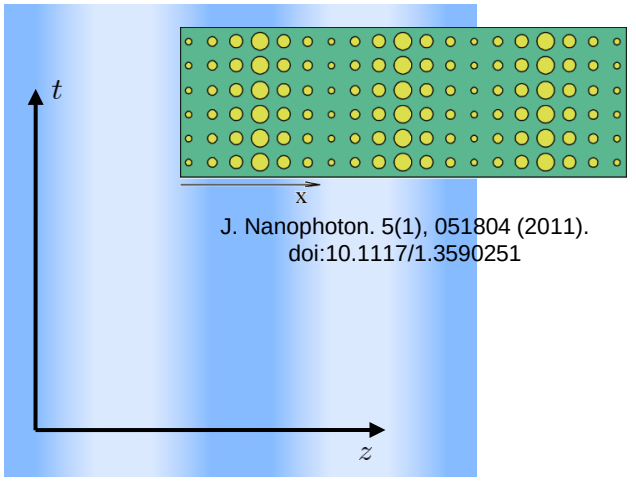


Half-space



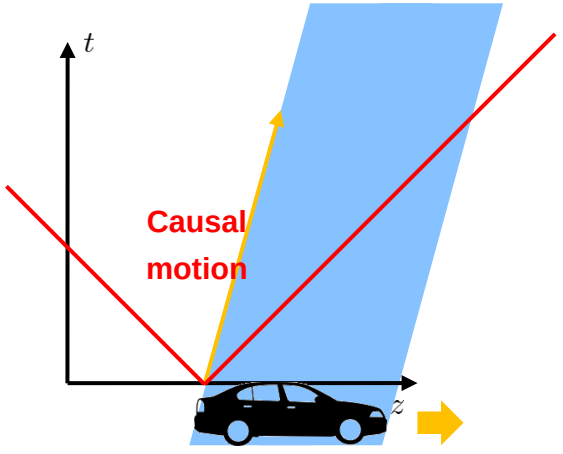
Advances in Photonic Crystals. ISBN 978-953-51-0954-9

Photonic crystal

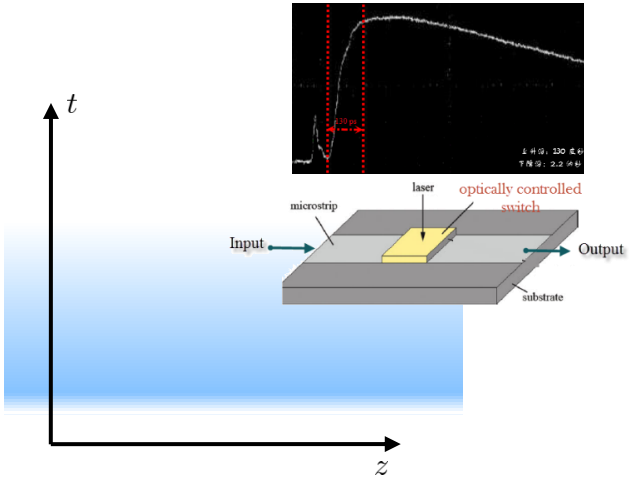


J. Nanophoton. 5(1), 051804 (2011). doi:10.1117/1.3590251

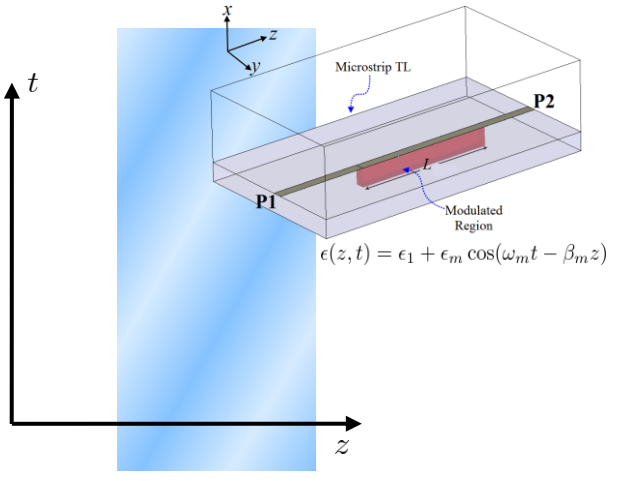
Spatially modulated medium



Moving structure



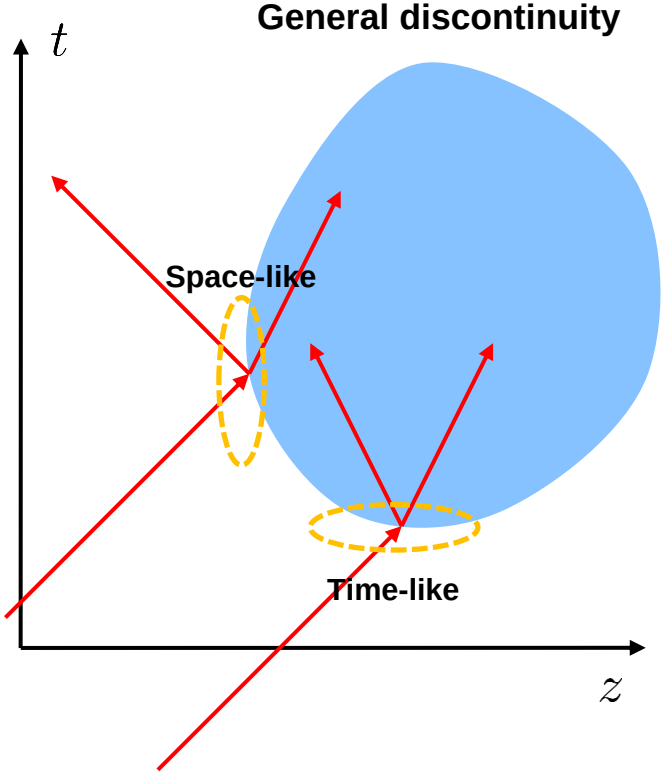
Time-varying medium
(hump-switched)

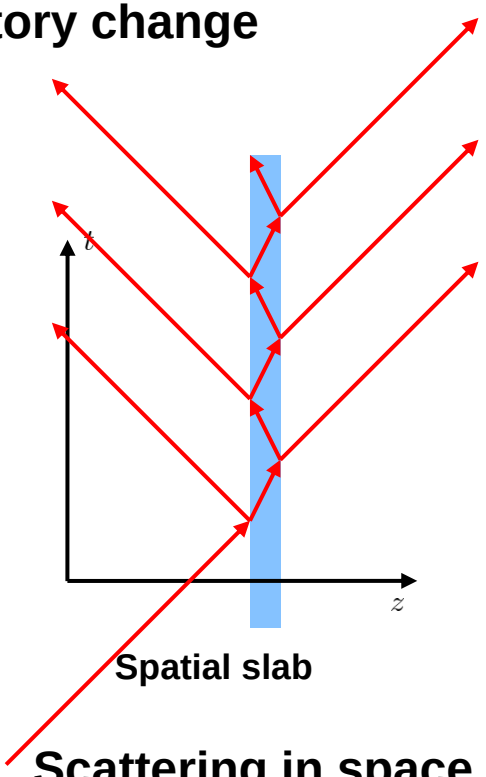


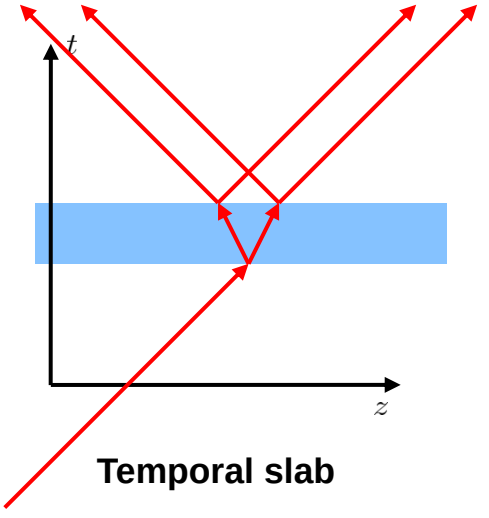
Space-time modulated structure

Scattering in Space and Scattering in Time

Trajectory change







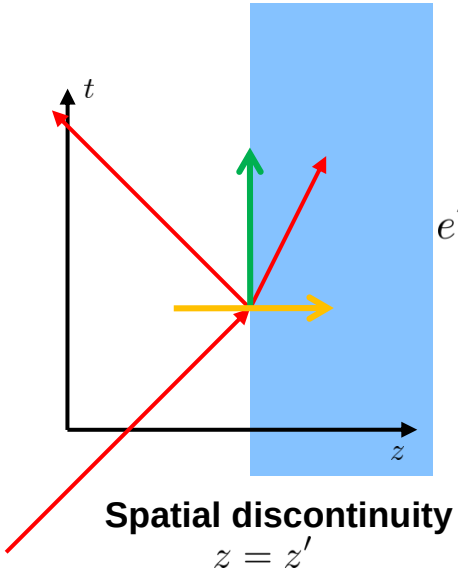
Scattering in space

Scattering in time

M. A. Salem and C. Caloz, "Scattering in Spatiotemporal Media," in IEEE AP-S Int. Antennas Propag. (APS), Vancouver, BC, Jul. 2015.

Images: various public domain sources

Space and Time Discontinuities



Spatial discontinuity

$z = z'$

Phase matching $\forall t \rightarrow \omega_2 = \omega_1$

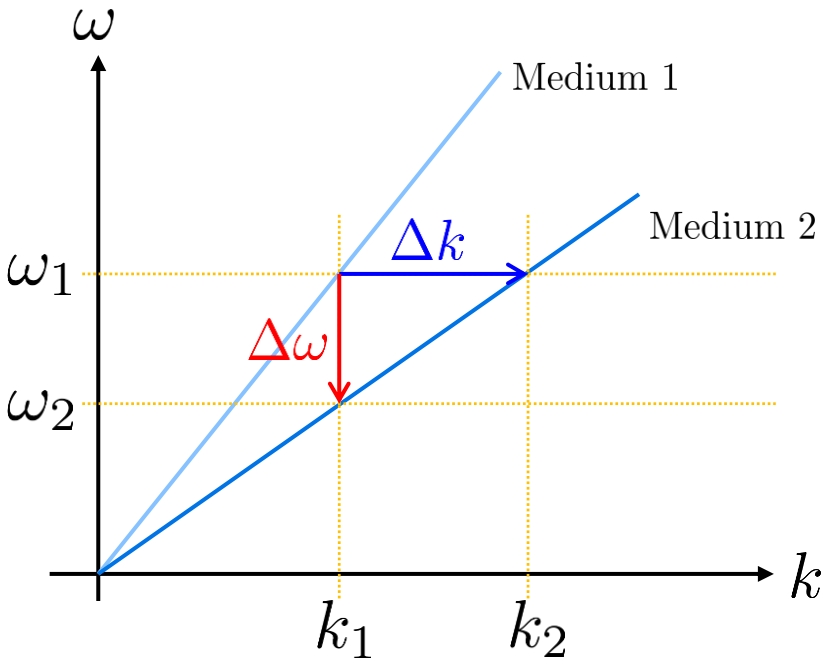
Continuity of flow

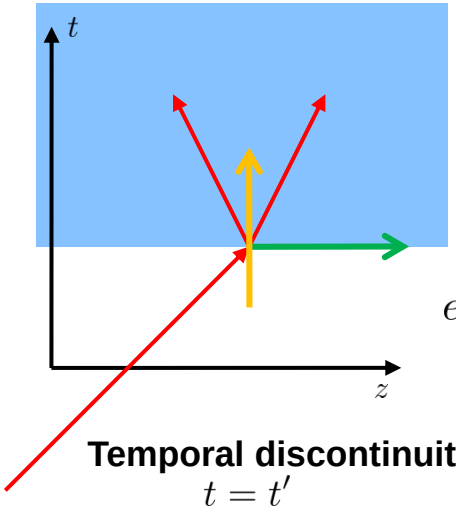
$$e^{ik_1 z'} e^{-i\omega_1 t} + \gamma e^{-ik_1 z'} e^{-i\omega_1 t} = \tau e^{ik_2 z'} e^{-i\omega_2 t}$$

$$1 + \gamma = \tau$$

$$k_1 - k_1\gamma = k_2\tau$$

Frequency shift at discontinuities





Temporal discontinuity

$t = t'$

Phase matching $\forall z \rightarrow k_2 = k_1$

Continuity of flow

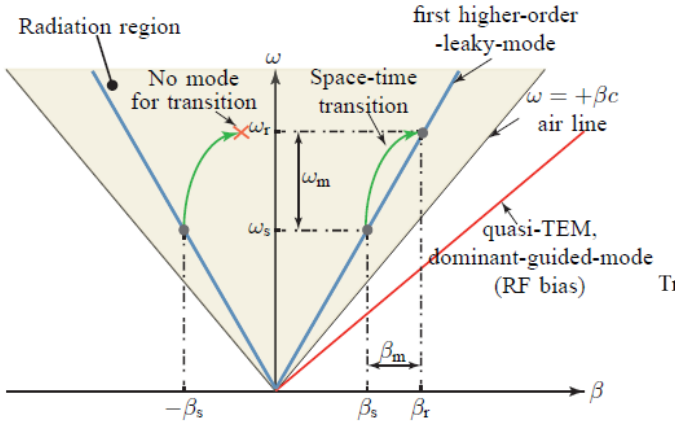
$$e^{ik_1 z} e^{-i\omega_1 t'} = \xi_f e^{ik_2 z} e^{-i\omega_2 t'} + \xi_b e^{ik_2 z} e^{i\omega_2 t'}$$

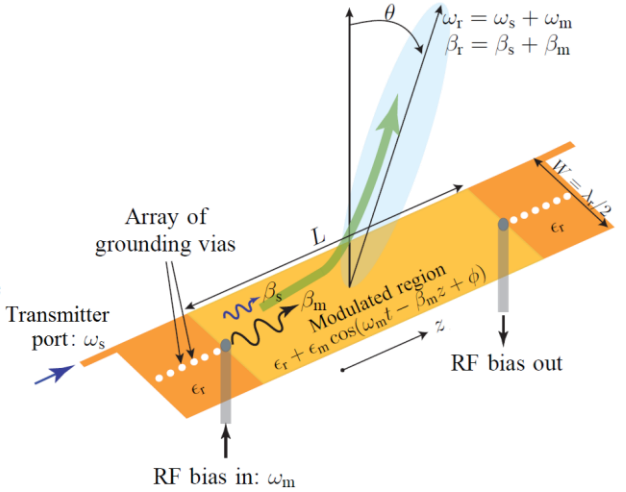
$$1 = \xi_f + \xi_b$$

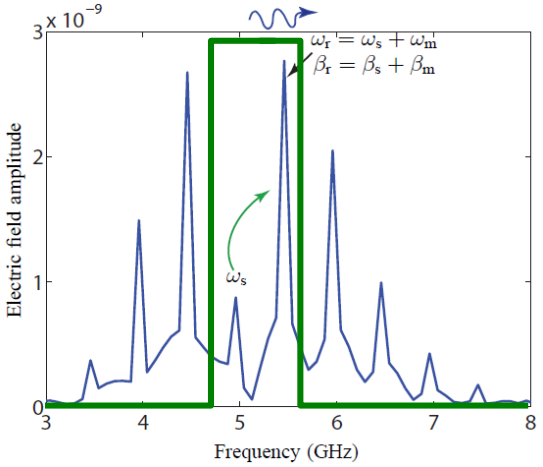
$$\omega_1 = \omega_2 \xi_f - \omega_2 \xi_b$$

M. A. Salem and C. Caloz, "Scattering in Spatiotemporal Media," in IEEE AP-S Int. Antennas Propag. (APS), Vancouver, BC, Jul. 2015.

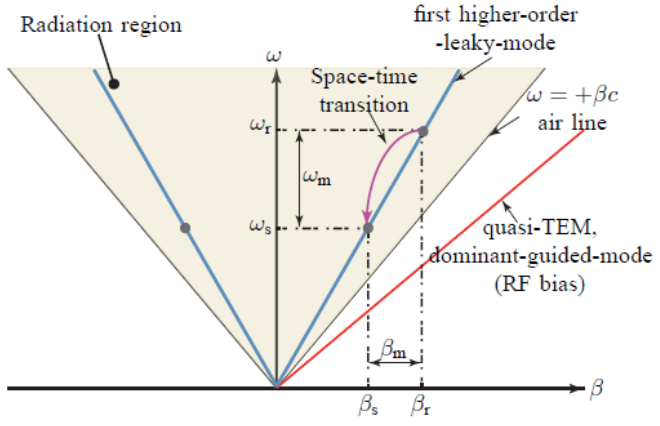
Space-Time Modulated NR Leaky-Wave Antenna

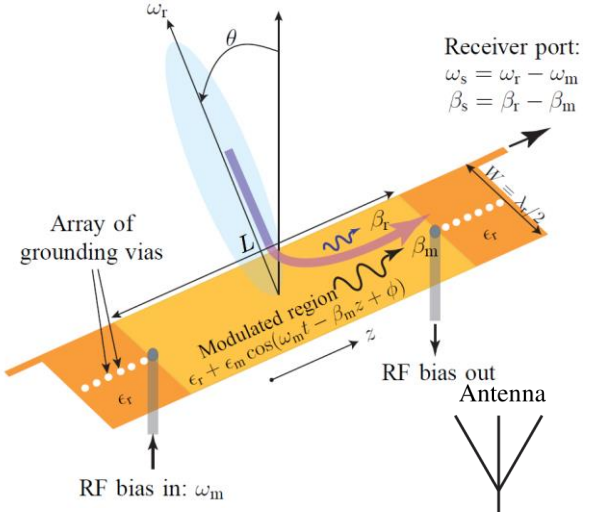


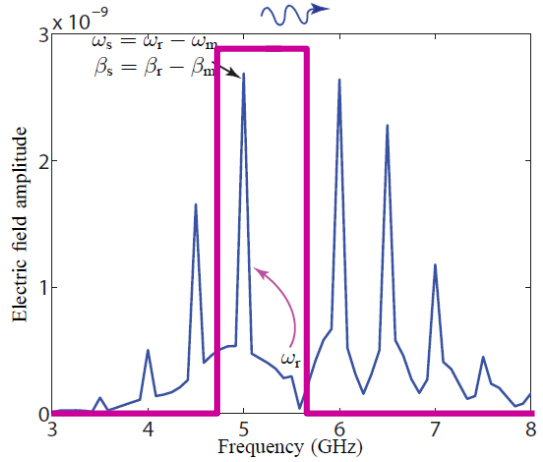




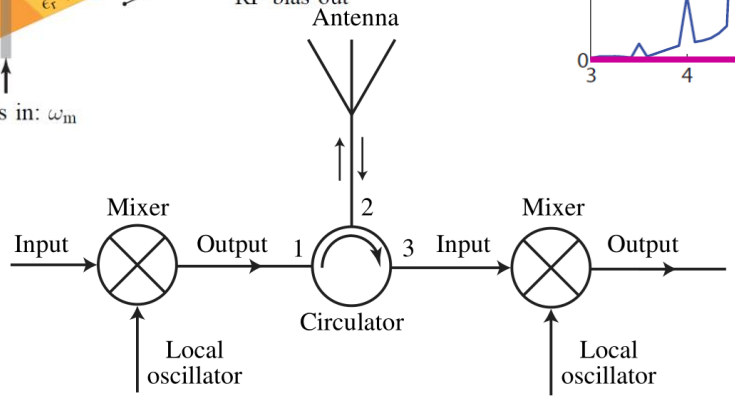
$$\epsilon = \epsilon_r + \epsilon_m \cos(\omega_m t - \beta_m z)$$







Equivalent System:



S. Taravati and C. Caloz," in *IEEE AP-S Int. Antennas Propag. (APS)*, Vancouver, BC, Jul. 2015.

The Space-Time DE – ST Processors QUIZ

1) What is the difference between the P-LWA RTSA and an optical grating ?

   THE EXCITATION, transmission line versus plane wave

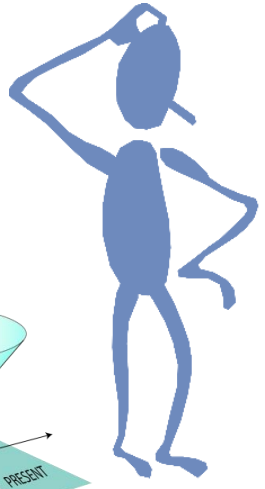
2) How many polychromatic metasurfaces have been reported so far ?

   ZERO. Trying the first for 2D real-time spectroscopy

3) Space-Time was discovered by Einstein.

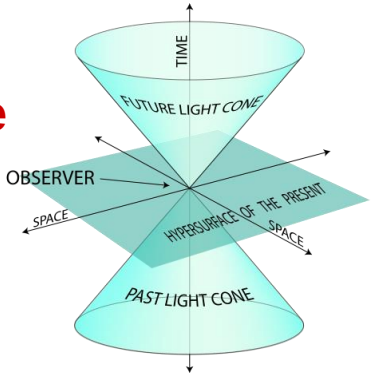
   NO, religion, philosophy, literature, mathematics

   Mostly Minkowski. Great for SR & used in GR



4) All the regions of Minkowsi's Space-Time are allowed.

   NO, only those in the PAST-FUTURE double cone



5) Is scattering in space and in time the same thing?

   YES, change of wave direction due to matter

   NO, time is only forward

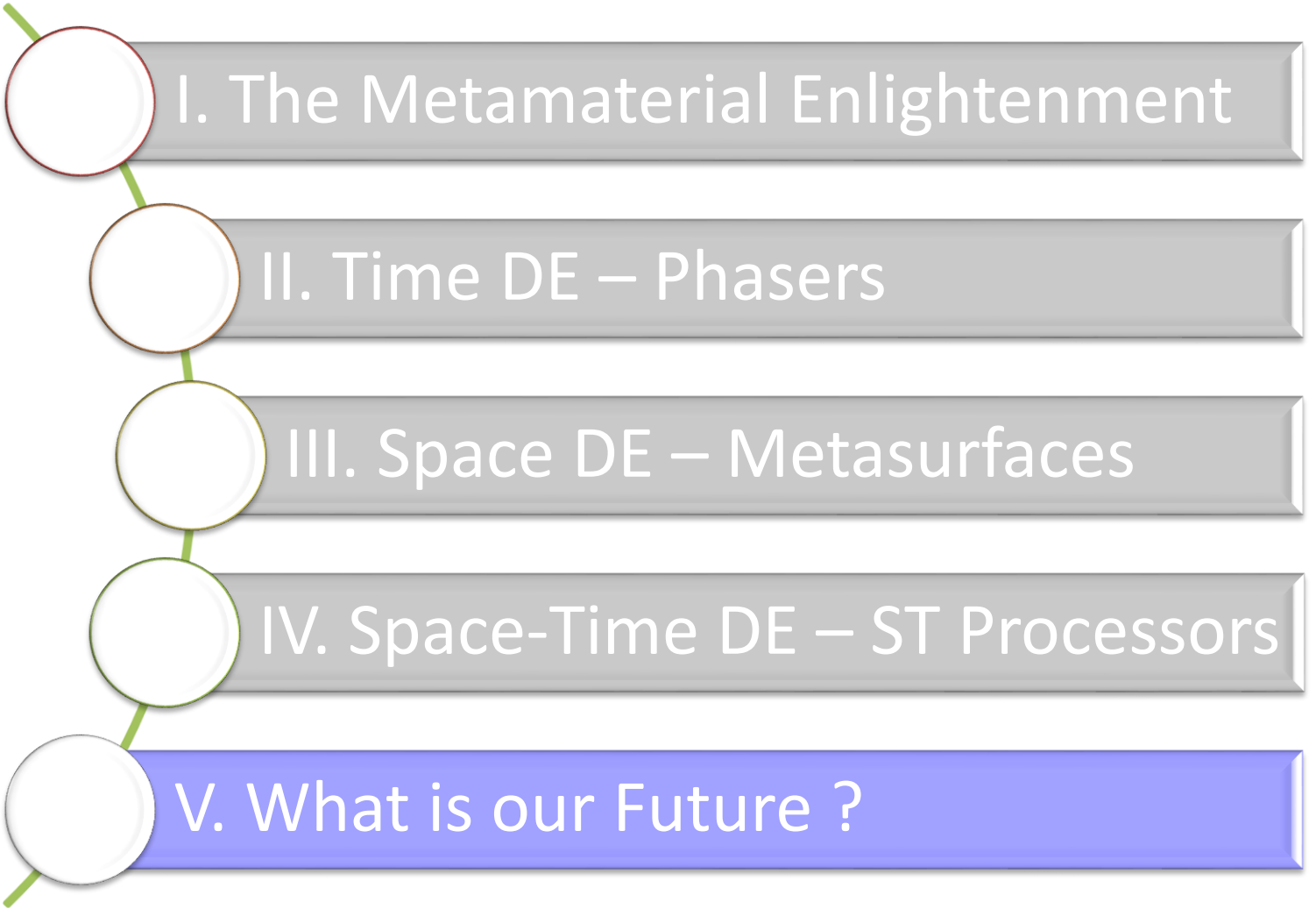
6) Space-Time discontinuities is an old field in electromagnetics.

   NOT THAT I KNOW. Very little literature. Perhaps plasma physics…

7) Space-Time structures are pure academic curiosities.

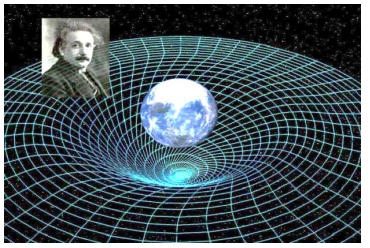
   NO, as a fundamental concept, potential for great applications

Outline

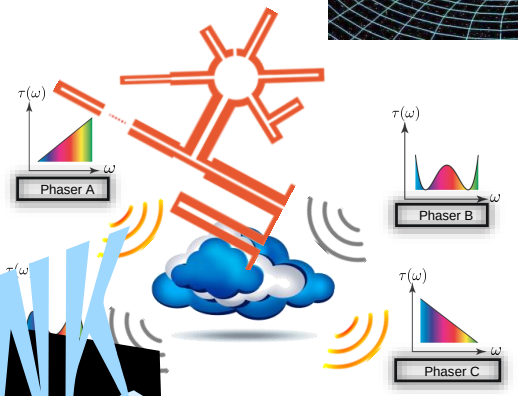

I. The Metamaterial Enlightenment
II. Time DE – Phasers
III. Space DE – Metasurfaces
IV. Space-Time DE – ST Processors
V. What is our Future ?

Lots of Fascinating EM Challenges Ahead

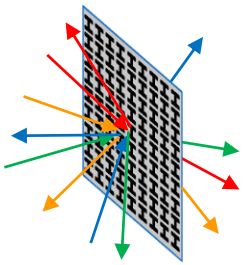
- **Time**, **Space**, **Space & Time** Processing
  → promising



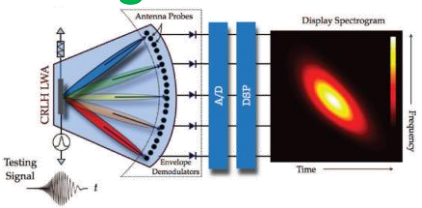
- Progress in **Time Processing**
  → **phasers**

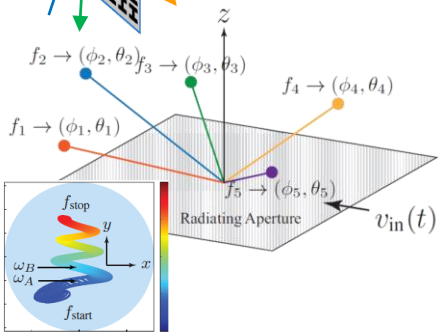
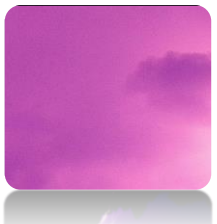


- Progress in **Space Proc**
  → **metasurfaces**



- Progress in **Space & Time Processing**
  → **combined structures**





- Exciting **spacetime opportunities**
  → **mixed discontinuities**